Solution:

Problem 1:

We can solve this by defining the infinitely repeating part of the expression with a variable and then solving for that variable.

Define the Repeating Part Let's call the infinite nested square root part of the expression **y**.

- $y = \sqrt{2 - \sqrt{2 - \sqrt{2 - \cdots}}}$

Set Up the Equation If you look inside the first square root, you'll see the exact same infinite expression, y, appears again.

- $y = \sqrt{2 - \cdots}$

Solve the Equation Now we can solve this equation for **y**.

- Square both sides: y^2 = 2 - y
- Rearrange it into a standard quadratic equation: y^2 + y - 2 = 0
- Factor the equation: (y + 2)(y - 1) = 0
- This gives two possible solutions: y = -2 or y = 1. Since **y** represents a square root, its value must be positive, so we choose **y = 1**.

Find the Final Value The original expression from the image is 2 - y. Since we found that y = 1, we can find the final answer.

2 - y = 2 - 1 = 1 💡

Problem 2:

We can find the area of the shaded rectangle by first using the given areas to determine the dimensions of each part of the larger rectangle.

Analyze the Square The top-right rectangle is a square with an area of **25**. Since the area of a square is side × side, its side length must be the square root of 25.

- Side length of the square = √25 = 5 meters.

- This tells us two things: the height of the top row is **5m**, and the width of the right column is **5m**.

Find the Width of the Left Rectangles The top-left rectangle has an area of **95** and shares a height of **5m** with the square. We can find its width:

- Width = Area / Height = 95 / 5 = 19 meters.
- This means the width of the entire left column is **19m**.

Find the Height of the Bottom Rectangles The bottom-left rectangle has an area of **228** and shares a width of **19m** with the one above it. We can find its height:

- Height = Area / Width = 228 / 19 = 12 meters.
- This means the height of the entire bottom row is **12m**.

Calculate the Shaded Area The shaded rectangle is in the bottom-right corner. We now know its dimensions:

- Its width is the same as the right column's width: **5m**.
- Its height is the same as the bottom row's height: **12m**.
- Area = Width × Height = 5 × 12 = 60 square meters. 🖼️

Problem 3:

The Pattern in the Grid

The pattern in this grid is that each number is the product of its **row header** and its **column header**. You can think of the grid as a multiplication table. Let's call the hidden numbers that define the rows **R1, R2, R3, R4** and the columns **C1, C2, C3, C4**. The number in any cell is the product of its corresponding R and C value.

Find the Headers We can figure out the hidden header numbers by looking at the rows and columns.

Row 1 (?, 8, 12, 7): For these numbers to be multiples of a single header number (**R1**), the header **R1** must be a common divisor of 8, 12, and 7. The only common divisor is 1. So, **R1 = 1**. This tells us that the headers for columns 2, 3, and 4 must be **C2=8**, **C3=12**, and **C4=7**.

Column 3 (12, 48, ?, 120): We know **C3 = 12**. We can find the row headers by dividing:

  - R1 = 12 / 12 = 1

- R2 = 48 / 12 = 4
- R4 = 120 / 12 = 10

Row 3 (?, 56, ?, ?): We use the value 56 and the header **C2 = 8** to find **R3**:

- R3 = 56 / 8 = 7

Column 1 (?, 4, ?, 10): We use the value 4 and the header **R2 = 4** to find **C1**:

- C1 = 4 / 4 = 1

So, our headers are:

Row Headers: **{1, 4, 7, 10}**

Column Headers: **{1, 8, 12, 7}**

Calculate the Missing Numbers Now we can fill in all the question marks by multiplying the corresponding headers.

- Top-left: R1 × C1 = 1 × 1 = 1
- Middle-left (row 2): R2 × C2 = 4 × 8 = 32
- Middle-left (row 3): R3 × C1 = 7 × 1 = 7
- Middle-right (row 3): R3 × C3 = 7 × 12 = 84
- Middle-right (row 3): R3 × C4 = 7 × 7 = 49
- Bottom-right: R4 × C4 = 10 × 7 = 70

Find the Sum The six missing numbers are **1, 32, 7, 84, 49, and 70**. Add them together:

- 1 + 32 + 7 + 84 + 49 + 70 = 243 🔢

Problem 4:

Simplify the Operation First, let's look at the definition of the operation: $A * B = A^2 + B^2 - 2 \cdot A \cdot B$. This is a perfect square trinomial, which can be factored and simplified as:

- A * B = (A - B)^2 This simplified rule makes the calculations much easier.

Calculate the First Part: (2 * 4) Using the simplified rule, we can calculate the value inside the first parenthesis.

- 2 * 4 = (2 - 4)^2 = (-2)^2 = 4

Calculate the Second Part: (6 * 8) Next, we calculate the value inside the second parenthesis.

- 6 * 8 = (6 - 8)^2 = (-2)^2 = 4

Calculate the Final Expression Now we can substitute the results from the first two steps back into the main expression.

- The expression is (2 * 4) * (6 * 8).
- This becomes 4 * 4.
- Using our rule one last time: 4 * 4 = (4 - 4)^2 = 0^2 = 0

The final value is **0**. 🔢

Problem 5:

Count the Two-Digit Palindromes A two-digit palindrome must have the form **aa**, where the first digit equals the second.

- The first digit, **a**, cannot be 0, so there are 9 choices for it (1 through 9).
- The second digit is determined by the first.
- This gives us 9 possible numbers (11, 22, 33, ..., 99).
- Total two-digit palindromes = **9**.

Count the Five-Digit Palindromes A five-digit palindrome must have the form **abcba**.

- The first digit, **a**, cannot be 0, so there are **9 choices** (1 through 9).
- The second digit, **b**, can be any digit from 0 to 9 (**10 choices**).
- The third digit, **c**, can be any digit from 0 to 9 (**10 choices**).
- The fourth and fifth digits are determined by the first and second digits.
- Total five-digit palindromes = 9 × 10 × 10 = 900.

Find the Ratio Now, we form the ratio of the two counts.

- Ratio = (Two-digit palindromes) : (Five-digit palindromes)
- Ratio = 9 : 900
- Simplifying by dividing both sides by 9, we get **1:100**. 🔄

Problem 6:

To compare these numbers, the easiest method is to rewrite them so they share a **common exponent**.

Find a Common Exponent The exponents are 8, 12, and 24. The greatest common divisor of these numbers is 4. We can rewrite each number as a new base raised to the power of 4.

Rewrite Each Number

- **2^24:** We can write this as (2^6)^4. Since 2^6 = 64, this becomes **64^4**.
- **10^8:** We can write this as (10^2)^4. Since 10^2 = 100, this becomes **100^4**.
- **5^12:** We can write this as (5^3)^4. Since 5^3 = 125, this becomes **125^4**.

Compare the New Numbers Now we just need to compare 64^4, 100^4, and 125^4. Since they all have the same exponent, we can simply compare their bases:

- 64 < 100 < 125

Determine the Final Order Based on the comparison of the bases, we can order the original numbers:

- 64^4 < 100^4 < 125^4
- This translates back to **2^24 < 10^8 < 5^12**. 🔢

Problem 7:

The digit in the millionth place is **1**.

To find the digit, we first need to figure out which number in the sequence (e.g., a 4-digit number, a 5-digit number, etc.) contains the 1,000,000th digit. We can do this by counting the total digits used for each group of numbers.

Count Digits from 1- to 5-Digit Numbers Let's find the total length of the string up to the last 5-digit number.

- **1-digit numbers (1-9):** 9 numbers × 1 digit = 9 digits
- **2-digit numbers (10-99):** 90 numbers × 2 digits = 180 digits
- **3-digit numbers (100-999):** 900 numbers × 3 digits = 2,700 digits

- **4-digit numbers (1000-9999):** 9,000 numbers × 4 digits = 36,000 digits
- **5-digit numbers (10000-99999):** 90,000 numbers × 5 digits = 450,000 digits

The total number of digits for all numbers from 1 to 99,999 is:

- 9 + 180 + 2,700 + 36,000 + 450,000 = 488,889 digits.

Locate the Position Among 6-Digit Numbers The millionth digit must be part of a 6-digit number. To find its exact position within the block of 6-digit numbers, we subtract the digits we've already accounted for.

- 1,000,000 - 488,889 = 511,111
- So, we are looking for the 511,111th digit after the sequence starts writing 6-digit numbers.

Find the Target Number Since each 6-digit number contributes 6 digits to the sequence, we can find which number contains our digit by dividing.

- 511,111 ÷ 6 = 85,185 with a **remainder of 1**.
- This result tells us that the digit is the **1st digit** (from the remainder) of the number that comes *after* the first 85,185 six-digit numbers have been written. We are looking for the 85,186th six-digit number.

Identify the Digit The first 6-digit number is 100,000. To find the 85,186th one:

- 100,000 + 85,186 - 1 = 185,185
- The number containing our digit is **185,185**.
- Since the remainder was 1, we need the **1st digit** of this number.

The first digit of 185,185 is **1**. 🔢

Problem 8:

Find the Number of Oranges First, we need to determine how many oranges were in the basket.

- The ratio of apples to oranges is **3:2**.
- This means the total number of "parts" in the ratio is 3 + 2 = 5.
- The total number of fruits is **40**. To find the value of one part, we divide the total fruits by the total parts: 40 ÷ 5 = 8 fruits per part.

- Since the oranges represent **2** parts of the ratio, the number of oranges is 2 × 8 = 16.

Calculate the Number of Bananas Ginny adds twice as many bananas as there are oranges.

- Number of Bananas = 2 × Number of Oranges
- Number of Bananas = 2 × 16 = 32 🍌

Problem 9:

Understand the Goal To find the value of the final expression, we first need to figure out which numbers (2, 3, 4, and 5) are assigned to which letters (A, B, C, and D). The rule is that the assignment must make the expression A^B - C^D as large as possible. To do this, we need to make A^B as large as possible and C^D as small as possible.

Maximize A^B Using the numbers {2, 3, 4, 5}, let's find the largest possible value for A^B. We should test the larger numbers in the base and exponent positions.

- 5^4 = 625
- 4^5 = 1024 The largest possible value is 1024, so we must have **A = 4** and **B = 5**.

Minimize C^D The numbers left to assign to C and D are {2, 3}. We need to find the smallest possible value for C^D.

- 2^3 = 8
- 3^2 = 9 The smallest possible value is 8, so we must have **C = 2** and **D = 3**.

Calculate the Final Expression Now we have the value for each letter:

- **A = 4**
- **B = 5**
- **C = 2**
- **D = 3**

Finally, we can calculate the value of (A + B) - (C - D):

- (4 + 5) - (2 - 3)
- (9) - (-1)
- 9 + 1 = 10 🧠

Problem 10:

Set Up the Equations Let's use variables to represent the number of each type of coin:

- **n** = number of 5-cent coins
- **d** = number of 10-cent coins
- **q** = number of 25-cent coins

Based on the problem, we can create three equations:

1. $n + d + q = 32$ (The total number of coins)
2. $5n + 10d + 25q = 390$ (The total value in cents)
3. $d = n + 4$ (The relationship between dimes and nickels)

Simplify the System We can use the third equation to substitute $n + 4$ for d in the first two equations. This will leave us with two equations and two variables.

- **Substituting into Equation 1:** $n + (n + 4) + q = 32$ $2n + q = 28$
- **Substituting into Equation 2:** $5n + 10(n + 4) + 25q = 390$ $5n + 10n + 40 + 25q = 390$ $15n + 25q = 350$, which simplifies to $3n + 5q = 70$ (by dividing by 5).

Solve for the Variables Now we solve the new system of two equations:

- $2n + q = 28$
- $3n + 5q = 70$

From the first equation, we can write $q = 28 - 2n$. Now substitute this into the second equation:

- $3n + 5(28 - 2n) = 70$
- $3n + 140 - 10n = 70$
- $-7n = -70$
- $n = 10$

Since there are **10** 5-cent coins, we can now find the number of 25-cent coins:

- $q = 28 - 2n$
- $q = 28 - 2(10)$
- $q = 28 - 20 = 8$

Marina has **8** 25-cent coins. 🪙

Problem 11:

Rewrite the Equation The key is to recognize that the largest number, ABCDEF, contains the other numbers. We can express ABCDEF as 1000 * ABC + DEF. This allows us to rewrite the original equation:

- A + BC + DEF + (1000 * ABC + DEF) = 343165
- Combining like terms, we get: 1000 * ABC + 2*DEF + BC + A = 343165

Estimate the Value of ABC The term 1000 * ABC is the largest part of the sum. The other terms (2*DEF + BC + A) are much smaller (their sum will be less than 2100). This means 1000 * ABC must be very close to the total of 343,165.

- ABC must be close to 343165 / 1000, which is approximately 343.
- If ABC = 343, then A=3 and C=3, but the problem states all digits must be **different**.
- Let's try the next closest integer, ABC = 342. This gives us A=3, B=4, and C=2, which are all different.

Solve for DEF Now we can substitute the values A=3, B=4, and C=2 (which also means BC=42 and ABC=342) into our rewritten equation:

- 1000 * (342) + 2*DEF + (42) + (3) = 343165
- 342000 + 2*DEF + 45 = 343165
- 342045 + 2*DEF = 343165
- 2*DEF = 343165 - 342045
- 2*DEF = 1120
- DEF = 1120 / 2
- DEF = 560

Verify the Solution The digits we found are A=3, B=4, C=2, D=5, E=6, F=0. These are all different digits. Let's check the original sum:

- A + BC + DEF + ABCDEF = 3 + 42 + 560 + 342560 = 343165. The sum is correct. 🧠

Problem 12:

Understand the Setup When students are evenly spaced in a circle, "directly across" means there is an equal number of students on the arc between them on both the left and the right sides.

Calculate the Difference The number of students that make up exactly half of the circle can be found by taking the difference between the numbers of the two students who are opposite each other.

- 17 - 3 = 14

Find the Total This difference of **14** represents half the students in the circle. To find the total number of students, simply double this amount.

- 14 × 2 = 28

Therefore, there are **28** students in the class. 👨‍👩‍👧‍👦

Problem 13:

Find the Prime Factorization First, we need to break down 2024 into its prime factors.

- Start by dividing by the smallest prime, 2. 2024 ÷ 2 = 1012 1012 ÷ 2 = 506
  506 ÷ 2 = 253
- Now we need to factor 253. It's not divisible by 2, 3, or 5. Let's try the next primes. 253 ÷ 7 = 36.14... 253 ÷ 11 = 23
- Both 11 and 23 are prime numbers.
- So, the full prime factorization of 2024 is **2 × 2 × 2 × 11 × 23**.

Sum the Distinct Prime Factors The distinct (unique) prime factors are 2, 11, and 23. Now, we just add them together.

- 2 + 11 + 23 = 36 ✅

Problem 14:

The sum of the digits is **17**.

Understand the Condition If a number N has the same remainder r when divided by 33 and 21, it means that N - r is perfectly divisible by both 33 and 21. In other words, N - r must be a common multiple of 33 and 21.

Find the Least Common Multiple (LCM) To find the common multiples, we first need the smallest one, the LCM.

- Prime factors of 33 are **3 × 11**.
- Prime factors of 21 are **3 × 7**.
- The LCM is 3 × 7 × 11 = 231. So, N - r must be a multiple of **231**.

Find the Largest Three-Digit Number We are looking for the **greatest three-digit number** N. This means N must be close to 999.

- Let's find the largest multiple of 231 that is less than 999: 231 × 4 = 924 *(231 × 5 = 1155, which is too large).*
- This means N - r = 924. To find N, we have N = 924 + r.
- To make N as large as possible, we need to use the largest possible remainder r. The remainder r must be smaller than the smallest divisor (21). The largest possible integer remainder is **20**.
- N = 924 + 20 = 944. So, the greatest three-digit number with this property is **944**.

Calculate the Sum of the Digits The question asks for the sum of the digits of this number.

- 9 + 4 + 4 = 17 🧠

Problem 15:

Set Up the Equations Let's represent the amount of money each person has with a letter: A for Anne, B for Beate, and C for Cecilie. Based on the problem, we can write three equations:

1. A + B = 120
2. B + C = 60
3. A + C = 70

Combine the Equations The quickest way to find the total is to add all three equations together:

- (A + B) + (B + C) + (A + C) = 120 + 60 + 70
- 2A + 2B + 2C = 250

Find the Total Now we can factor out the 2 from the left side of the equation:

- 2 * (A + B + C) = 250 To find the total sum A + B + C, just divide both sides by 2:
- A + B + C = 250 / 2
- A + B + C = 125 💰