Solution:

Problem 1:

Count the Perfect Squares We need to find how many perfect squares are between 200 and 700.

- $14^2 = 196$(This is too small).
- $15^2 = 225$(This is the first one).
- $26^2 = 676$(This is the last one).
- $27^2 = 729$(This is too large).
- The perfect squares are 15^2, $16^2, \ldots, 26^2$. To count them, we do 26-15+1=12.
- There are 12 perfect squares.

Count the Perfect Cubes Next, we find how many perfect cubes are between 200 and 700.

- $5^3 = 125$(This is too small).
- $6^3 = 216$
- $7^3 = 343$
- $8^3 = 512$
- $9^3 = 729$(This is too large).
- There are 3 perfect cubes.

Check for Overlap A number that is both a perfect square and a perfect cube must be a perfect 6th power. There are **no** numbers in the range of 200 to 700 that are perfect 6th powers, so there is no overlap between the two groups.

Calculate the Total Since there is no overlap, we can simply add the counts together.

- 12 (squares) + 3 (cubes) = 15
- There are **15** such integers in total. 🔢

Problem 2:

Calculate the Travel Time First, we need to find out how long his trip took. We can use the formula Time = Distance / Speed.

- Distance = 60 km
- Speed = 80 km/h
- Time = 60 km / 80 km/h = 0.75 hours To make this easier to work with, let's convert the time to minutes:
- 0.75 hours × 60 minutes/hour = 45 minutes
- The trip took **45 minutes**.

Determine the Arrival Time His appointment was at 10:00 am, but he arrived 20 minutes late.

- Arrival Time = 10:00 am + 20 minutes = 10:20 am

Find the Departure Time To find when he left, subtract the travel time from his arrival time.

- Departure Time = 10:20 am - 45 minutes
- 10:20 am - 20 minutes = 10:00 am
- 10:00 am - 25 minutes = 9:35 am
- Stephen left home at **9:35 am**. ⏰

Problem 3:

Understand the Geometric Principle When a line is drawn from a vertex of a triangle to its opposite base (like the line AD in triangle ABC), it splits the triangle into two smaller triangles (ADC and ABD). These two smaller triangles share the same height. Because they share the same height, the **ratio of their areas is the same as the ratio of their bases**.

Apply the Ratio

  - We are given that the ratio of the bases is DC:BD = 5:3.
  - This means the ratio of the areas must also be Area(ADC) : Area(ABD) = 5:3.
  - The total "parts" in this ratio is 5 + 3 = 8.

Calculate the Area The total area of the large triangle ABC is **24** square units, and this area is split into 8 parts according to the ratio. We want to find the area of triangle ADC, which corresponds to **5** of those parts.

- Area(ADC) = (5/8) × Total Area
- Area(ADC) = (5/8) × 24
- Area(ADC) = 5 × (24 / 8)
- Area(ADC) = 5 × 3 = 15

The area of triangle ADC is **15** square units. 📐

Problem 4:

Analyze the Division Property The problem states that when 109 is divided by the two-digit number ab, the remainder is 4. This means that the number ab must be a divisor of 109 - 4.

- 109 - 4 = 105 So, the number ab is a divisor of **105**.

Find the Divisors Let's list all the divisors of 105.

- The divisors are 1, 3, 5, 7, 15, 21, 35, and 105.

Apply the Given Conditions Now we filter this list using the conditions from the problem:

- The number ab must be a **two-digit** integer. This narrows our list down to **{15, 21, 35}**.
- The number ab is **not divisible by 5**. This eliminates 15 and 35.

Determine the Number and the Sum of its Digits The only number that satisfies all the conditions is **21**.

- For the number 21, the tens digit is a = 2 and the ones digit is b = 1.
- The sum of the digits is a + b = 2 + 1 = 3. ✅

Problem 5:

Find the Magic Constant The magic constant is **39**. (3 × the median number, 13).

Find the Center Cell and the Cell Opposite '5' The center cell is **13**. The cell opposite the 5 (in the middle column) is 39 - 5 - 13 = 21.

Set Up Equations Let's call the top-left corner **A** and the top-right corner **B**.

- **First Column:** A + 9 + X = 39 → A = 30 - X
- **Top Row:** A + 5 + B = 39
- **Diagonal (Top-Right to Bottom-Left):** B + 13 + X = 39 → B = 26 - X

Solve for X Now, substitute the expressions for A and B into the top row equation:

- (30 - X) + 5 + (26 - X) = 39
- 61 - 2X = 39
- 61 - 39 = 2X
- 22 = 2X
- X = 11

The number in the place of X is **11**. ✅

Problem 6:

To find the total, we can count the number of valid integers for each length (1-digit, 2-digit, and 3-digit) and add them together.

Identify the Allowed Digits The problem asks for numbers written *without* the digits 7, 8, or 9. This means we can only use the digits from the set **{0, 1, 2, 3, 4, 5, 6}**, which is a total of **7** allowed digits.

Count the 1-Digit Numbers The positive integers we can form are {1, 2, 3, 4, 5, 6}.

- This gives us **6** possible numbers.

Count the 2-Digit Numbers

- For the **first digit**, we can't use 0, so we have 6 choices {1, 2, 3, 4, 5, 6}.
- For the **second digit**, we can use any of the 7 allowed digits {0, 1, 2, 3, 4, 5, 6}.
- Total 2-digit numbers = 6 × 7 = 42

Count the 3-Digit Numbers

- For the **first digit**, we have 6 choices (it can't be 0).

- For the **second digit**, we have 7 choices.
- For the **third digit**, we have 7 choices.
- Total 3-digit numbers = $6 \times 7 \times 7 = 294$

Calculate the Total Finally, we add the counts from all the groups together.

- 6 (1-digit) + 42 (2-digit) + 294 (3-digit) = 342

Problem 7:

To solve this, we need to use the divisibility rules for 72. For a number to be divisible by 72, it must be divisible by both **8** and **9** (since $8 \times 9 = 72$ and they are coprime).

Divisibility by 8 A number is divisible by 8 if its last three digits form a number that is divisible by 8. In this case, the number formed by the last three digits is **94B**.

- We need to find a digit B such that 94B is a multiple of 8.
- We can test values or use division. Let's check nearby multiples of 8: $8 \times 117 = 936$, $8 \times 118 = 944$, $8 \times 119 = 952$.
- The only number that matches the form 94B is 944.
- Therefore, **B = 4**.

Divisibility by 9 A number is divisible by 9 if the sum of its digits is a multiple of 9. Now that we know B = 4, our six-digit number is **47A944**.

- Let's sum the digits: $4 + 7 + A + 9 + 4 + 4 = 28 + A$.
- The sum $28 + A$ must be a multiple of 9. The next multiple of 9 after 28 is 36.
- $28 + A = 36$
- $A = 36 - 28 = 8$
- Therefore, **A = 8**.

Calculate A – B Now we can find the value of A – B.

- A - B = 8 - 4 = 4 ✅

Problem 8:

We can find the total by counting the number of palindromes for each length (1-digit, 2-digit, and 3-digit) and then adding them together.

1-Digit Palindromes All single-digit positive integers (1, 2, 3, 4, 5, 6, 7, 8, 9) are palindromes because they read the same forwards and backward.

- Total 1-digit palindromes: **9**

2-Digit Palindromes A two-digit number is a palindrome if both digits are the same (e.g., 11, 22). The first digit cannot be 0.

- The possibilities are 11, 22, 33, 44, 55, 66, 77, 88, and 99.
- Total 2-digit palindromes: **9**

3-Digit Palindromes A three-digit number is a palindrome if the first and last digits are the same (e.g., 101, 242).

- The **first digit** can be any number from 1 to 9 (**9 choices**).
- The **middle digit** can be any number from 0 to 9 (**10 choices**).
- The **last digit** must be the same as the first digit (**1 choice**).
- Total 3-digit palindromes = $9 \times 10 \times 1 = 90$

Total Count Finally, add the counts from all three groups together.

- 9 (1-digit) + 9 (2-digit) + 90 (3-digit) = 108

Problem 9:

To find the number of solutions, we can find the unique sets of numbers that meet the conditions and then count the different ways to arrange them.

Analyze the Constraints We need to find ordered triplets (x, y, z) where each number is an integer from 0 to 6, and $x + y + z = 16$. Since the maximum value for any of the integers is 6, their maximum possible sum is $6 + 6 + 6 = 18$. Because our target sum of 16 is very close to this maximum, we know the numbers must be large.

Find the Possible Sets of Numbers Let's find the combinations of three numbers that add up to 16.

- **Case 1:** What if two of the numbers are 6? $6 + 6 + z = 16$ $12 + z = 16$ $z = 4$ This gives us the set of numbers **{6, 6, 4}**. All numbers in this set are within the 0-6 range.

- **Case 2:** What if one number is 6 and another is 5? 6 + 5 + z = 16 11 + z = 16 z = 5 This gives us the set of numbers **{6, 5, 5}**. All numbers in this set are also valid.

(No other combinations are possible. For example, if the largest number were 5, the maximum sum would be 5+5+5=15, which is too small.)

Count the Different Arrangements (Solutions) Now we just need to count the unique, ordered arrangements for each set.

- For the set **{6, 6, 4}**, the possible solutions are:
  1. (6, 6, 4)
  2. (6, 4, 6)
  3. (4, 6, 6)
- For the set **{6, 5, 5}**, the possible solutions are: 4. (6, 5, 5) 5. (5, 6, 5) 6. (5, 5, 6)

Adding these up, we have a total of 3 + 3 = 6 different solutions. 🧠

Problem 10:

Find the Discount Amount First, find the difference between the original price and the sale price to determine the discount in dollars.

- $150 (original) - $108 (sale) = $42
- The discount amount is **$42**.

Calculate the Percent Discount Next, divide the discount amount by the original price and multiply by 100 to get the percentage.

- (Discount Amount / Original Price) × 100%
- ($42 / $150) × 100% = 0.28 × 100% = 28%

The discount is **28%**. 💰

Problem 11:

Find the Pattern The problem states that the sum of any three consecutive boxes is always 2024. Let's label the first four boxes as Box 1, Box 2, Box 3, and Box 4.

- According to the rule: Box 1 + Box 2 + Box 3 = 2024
- And also: Box 2 + Box 3 + Box 4 = 2024

If you compare these two equations, you can see that **Box 1** must be equal to **Box 4**. This same logic applies to the entire sequence, creating a simple repeating pattern every three boxes.

- Box 1 = Box 4 = Box 7
- Box 2 = Box 5 = Box 8
- Box 3 = Box 6

Solve for the Leftmost Box The leftmost box is **Box 1**. Following the pattern we found, the number in Box 1 must be the same as the number in Box 7.

- The problem gives the number in the 7th box as **789**.
- Therefore, the number in the 1st box must also be **789**. ➡️

Problem 12:

Set Up the Equations Let's call the larger number **L** and the smaller number **S**. We can turn the information from the problem into two simple equations:

  - From "The difference of two numbers is 966": L - S = 966
  - From "When the larger number is divided by the smaller one, the quotient is 25 and the remainder is 6": L = 25 * S + 6

Solve the Equations Now we can solve for **S** by substituting the second equation into the first one. We'll replace **L** with the expression
(25 * S + 6):

  - (25 * S + 6) - S = 966

Simplify and solve for S:

  - 24 * S + 6 = 966
  - 24 * S = 960
  - S = 960 / 24
  - S = 40

The smaller number is **40**. ✅

Problem 13:

Understand the Ratio The problem states that the number is a two-digit number ab and the ratio of the digit **a** to the digit **b** is **1:2**. This can be written as the equation:

- b = 2a

Test Possible Values We can now test the possible values for the first digit, **a**. Remember, **a** cannot be 0 (since it's a two-digit number), and **b** cannot be greater than 9.

- If **a = 1**, then b = 2 × 1 = 2. The number is **12**.
- If **a = 2**, then b = 2 × 2 = 4. The number is **24**.
- If **a = 3**, then b = 2 × 3 = 6. The number is **36**.
- If **a = 4**, then b = 2 × 4 = 8. The number is **48**.
- If **a = 5**, then b = 2 × 5 = 10. This is not possible because **b** must be a single digit.

Count the Numbers The possible numbers that satisfy the condition are 12, 24, 36, and 48. Counting these up, we find there are **4** numbers in total. 🔢

Problem 14:

This problem can be solved with a little bit of algebra, and the answer will be the same no matter which number Alfred picks.

Set Up the Expressions Let's call the number Alfred picks **A**.

  - Brian's result, B, is 10 more than Alfred's number: B = A + 10
  - Christine's result, C, is 10 less than Alfred's number: C = A - 10

Calculate B – C Now, let's find the difference between B and C by substituting the expressions from above.

  - B - C = (A + 10) - (A - 10)
  - B - C = A + 10 - A + 10

The A and -A cancel each other out, leaving:

  - B - C = 10 + 10 = 20

The result is always **20**, regardless of the number Alfred chose. 💡

Problem 15:

Understand Trailing Zeros Trailing zeros in a base-10 number are created by factors of 10. Since the prime factorization of 10 is 2×5, the number of trailing zeros is determined by how many pairs of 2s and 5s exist in the number's prime factorization. This will be limited by whichever prime factor appears fewer times.

Find the Prime Factorization First, let's break down the original expression, $20^{50} \times 50^{20}$, into its prime factors (2s and 5s).

- For 20^{50}: $20 = 2 \times 2 \times 5 = 2^2 \times 5$ So, $20^{50} = (2^2 \times 5)^{50} = 2^{100} \times 5^{50}$
- For 50^{20}: $50 = 2 \times 5 \times 5 = 2 \times 5^2$ So, $50^{20} = (2 \times 5^2)^{20} = 2^{20} \times 5^{40}$

Combine the Factors Now, multiply the two parts together and combine the exponents for each prime factor.

- Total = $(2^{100} \times 5^{50}) \times (2^{20} \times 5^{40})$
- Total = $2^{(100 + 20)} \times 5^{(50 + 40)}$
- Total = $2^{120} \times 5^{90}$

Count the Zeros The complete prime factorization has 120 factors of 2 and 90 factors of 5. Since we need one of each to make a 10, the number of zeros is limited by the smaller exponent.

- The number of factors of 5 is **90**.
- Therefore, we can make 90 pairs of (2×5), which means the number will have **90** trailing zeros. 🔟