Solution:

Problem 1:

First, list all prime numbers less than 15. Prime numbers less than 15 are: 2, 3, 5, 7, 11, 13.

Now, for each of these prime numbers, apply the "super-prime" rule: double it and subtract 1. Then check if the result is also a prime number.

1. **For 2:** Doubling 2 gives $2\times2=4$. Subtracting 1 gives $4-1=3$. Is 3 a prime number? Yes. So, 2 is a super-prime.
2. **For 3:** Doubling 3 gives $2\times3=6$. Subtracting 1 gives $6-1=5$. Is 5 a prime number? Yes. So, 3 is a super-prime.
3. **For 5:** Doubling 5 gives $2\times5=10$. Subtracting 1 gives $10-1=9$. Is 9 a prime number? No (since $9=3\times3$). So, 5 is not a super-prime.
4. **For 7:** Doubling 7 gives $2\times7=14$. Subtracting 1 gives $14-1=13$. Is 13 a prime number? Yes. So, 7 is a super-prime.
5. **For 11:** Doubling 11 gives $2\times11=22$. Subtracting 1 gives $22-1=21$. Is 21 a prime number? No (since $21=3\times7$). So, 11 is not a super-prime.
6. **For 13:** Doubling 13 gives $2\times13=26$. Subtracting 1 gives $26-1=25$. Is 25 a prime number? No (since $25=5\times5$). So, 13 is not a super-prime.

The super-primes less than 15 are 2, 3, and 7. There are 3 such super-primes.

The correct answer is C) 3.

Problem 2:

Represent the Numbers Since the ratio of the three numbers is **1:2:4**, we can represent them algebraically using a common multiplier, **x**.

- First number: **x**
- Second number: **2x**
- Third number: **4x**

Set Up the Equation The problem states that the sum of their squares is **189**. We can write this as an equation:

- $x^2 + (2x)^2 + (4x)^2 = 189$

Solve for x Now, solve the equation to find the value of **x**.

- $x^2 + 4x^2 + 16x^2 = 189$
- $21x^2 = 189$
- $x^2 = 9$
- $x = 3$

Find the Three Numbers Use the value of **x = 3** to find the actual numbers.

- First number: x=3
- Second number: 2x=2×3=6
- Third number: 4x=4×3=12
- The three numbers are **3, 6, and 12**.

Calculate the Final Sum The question asks for the sum of these three numbers.

- 3+6+12=21 ✅

Problem 3:

We can find the area of the shaded rectangle by first determining its width and height using the dimensions of the larger rectangle and the surrounding smaller ones.

Find the Width of the Shaded Rectangle

The total width of the large rectangle is given by the labels on the bottom: 6 + 5 = 11 units.

The shaded rectangle is positioned horizontally between the top-left rectangle (width **4**) and the bottom-right rectangle (width **5**).

To find the width of the middle space it occupies, we can subtract the widths of the left and right rectangles from the total width: Width = 11 - 4 - 5 = 2 units.

Find the Height of the Shaded Rectangle

The total height of the large rectangle is given by the labels on the left: 5 + 1 = 6 units.

The shaded rectangle is positioned vertically between the top-right rectangle (height **3**) and the bottom-left rectangle (height **1**).

To find the height of the middle space it occupies, we can subtract the heights of the top and bottom rectangles from the total height: Height = 6 - 3 - 1 = 2 units.

Calculate the Area Now that we know the dimensions of the shaded rectangle are **2 by 2**, we can calculate its area:

Area = Width × Height

Area = 2 × 2 = 4 square units. ✅

Problem 4:

This problem involves a repeating pattern, and the key to solving it is to use division and find the remainder.

Identify the Pattern The sequence is a block of **5** numbers (3, 4, 5, 6, 7) that repeats over and over.

Use Division to Find the Position To find which number is in the 101st position, you can divide 101 by the length of the pattern (5).

101 ÷ 5 = 20 with a **remainder of 1**.

Interpret the Remainder The remainder tells you where you are in the cycle.

A remainder of 1 means it's the **1st** number in the pattern.

A remainder of 2 means it's the **2nd** number in the pattern.

...and so on. A remainder of 0 would mean it's the last number.

Since the remainder is **1**, the 101st number is the same as the **first** number in the sequence.

The first number is **3**. 🔄

Problem 5:

Tim bought 24 pens for $144. To find the cost of one pen, we divide the total cost by the number of pens.

- $144 ÷ 24 = $6
- Each pen costs **$6**.

We know that 4 pens cost the same as 6 mechanical pencils.

- Cost of 4 pens: 4 × $6 = $24

- This means 6 mechanical pencils also cost $24. To find the cost of one, we divide.
- $24 ÷ 6 = $4
- Each mechanical pencil costs **$4**.

Now we can find how many mechanical pencils Tim could buy with his $156 budget.

- $156 ÷ $4 = 39
- He could buy **39** mechanical pencils. ✏️

Problem 6:

The property is that a number must be divisible by its ones digit. We need to check every number from **21 to 39**.

Here are the numbers in that range that have the property:

- **21** (divisible by 1)
- **22** (divisible by 2)
- **24** (divisible by 4)
- **25** (divisible by 5)
- **31** (divisible by 1)
- **32** (divisible by 2)
- **33** (divisible by 3)
- **35** (divisible by 5)
- **36** (divisible by 6)

Counting these up, we find there are a total of **9** numbers. ✅

Problem 7:

The key to solving this is to first find the "magic constant" — the number that each row, column, and diagonal sums to.

1. **Find the Center Number (C)** In this magic square, we can find the center number **C** by comparing a column and a diagonal that don't contain C.

The sum of the first column is: A + 18 + 25

The sum of the top-right-to-bottom-left diagonal is: B + C + 25

Since all sums are equal, we can look at another pair.

Let's compare the first column (A + 18 + 25) with the main diagonal (A + C + 21). Since both sums must be the same: A + 18 + 25 = A + C + 21

We can cancel **A** from both sides: 18 + 25 = C + 21 43 = C + 21

Solving for C, we get: C = 43 - 21 = 22

2. **Find the Magic Constant** In a 3x3 magic square, the magic constant is always **3 times the center number**.

Magic Constant = 3 × C

Magic Constant = 3 × 22 = 66

3. **Find the Remaining Numbers** Now that we know every row, column, and diagonal must sum to **66**, we can easily find the other missing numbers.

D (from the middle row): 18 + C + D = 66 → 18 + 22 + D = 66 → 40 + D = 66 → **D = 26**

A (from the first column): A + 18 + 25 = 66 → A + 43 = 66 → **A = 23**

E (from the second column): 24 + C + E = 66 → 24 + 22 + E = 66 → 46 + E = 66 → **E = 20**

B (from the first row): A + 24 + B = 66 → 23 + 24 + B = 66 → 47 + B = 66 → **B = 19**

4. **Calculate the Final Sum** Now, add the values of the five erased numbers together:

A + B + C + D + E

23 + 19 + 22 + 26 + 20 = 110 ✅

Problem 8:

To solve this, we need to count the number of three-digit integers that meet all the given rules. The most important rules are that the number must be even (ending in 2 or 8) and between 200 and 700 (starting with 2 or 5).

Since the digit **2** is a possible choice for both the first and last digit (but can't be used twice), we can solve this by breaking it into two cases.

Case 1: The first digit is 2

Hundreds digit: Must be **2** (1 choice).

Ones digit: Must be an even number from the set {1, 2, 5, 7, 8, 9}. The even digits are 2 and 8. Since digits can't be repeated and we've already used 2, the ones digit must be **8** (1 choice).

Tens digit: We have used the digits 2 and 8. The remaining digits from the set are {1, 5, 7, 9}. This gives us **4 choices**.

The total count for this case is 1 × 4 × 1 = 4 numbers.

Case 2: The first digit is 5

Hundreds digit: Must be **5** (1 choice).

Ones digit: Must be an even number from the set. Both **2** and **8** are available (2 choices).

Tens digit: We have used the digit 5 and one of the even digits (2 or 8). This leaves **4 choices** for the middle digit.

The total count for this case is 1 × 4 × 2 = 8 numbers.

Total Count

Finally, add the results from both cases together to get the total number of integers Josh could write.

4 (from Case 1) + 8 (from Case 2) = 12 🔢

Problem 9:

There are **zero** ways to write 47 as the sum of two primes.

Explanation

1. **Odd and Even Numbers:** For two numbers to add up to an odd number (like 47), one number must be **even** and the other must be **odd**.
2. **Prime Numbers:** The only even prime number is **2**.
3. **The Only Possibility:** This means that if 47 were the sum of two primes, one of those primes would have to be 2.
    - 2 + ? = 47
4. **The Result:** Solving this gives us the other number:
    - 47 - 2 = 45

5. **Conclusion:** The number **45** is not a prime number (it's divisible by 3, 5, 9, etc.). Since this single possibility doesn't work, there are no ways to write 47 as the sum of two primes. 💡

Problem 10:

We can solve this problem with a bit of logical reasoning.

Calculate the Maximum Score If Arya had answered all 10 questions correctly, her score would have been:

- 10 questions × 5 points/question = 50 points

Find the Difference in Score Her actual score was 29, so the difference between the maximum possible score and her score is:

- 50 points - 29 points = 21 points

Determine the Impact of an Incorrect Answer Every time a correct answer is changed to an incorrect one, the score doesn't just go down by 5 points; it goes down by 7. This is because she loses the **5 points** she would have earned and gets an additional **2 points** deducted.

- 5 (lost points) + 2 (penalty points) = 7 points

Find the Number of Incorrect Answers To find out how many questions she answered incorrectly, we divide the total score difference by the point loss for each wrong answer.

- 21 points ÷ 7 points/question = 3 questions
- This means she answered **3** questions incorrectly.

Find the Number of Correct Answers Since she answered 10 questions in total:

10 total questions - 3 incorrect questions = 7 correct questions ✅

Problem 11:

Identify the Key Constraint For a number to be a multiple of 5, its last digit must be either 0 or 5. From the set of available digits {1, 2, 3, 4, 5, 6}, the only choice for the last digit is **5**.

Fix the Last Digit Since the last digit is fixed, we can set up the six positions for our number like this, with **1** choice for the final spot:

- _ _ _ _ _ 5

Arrange the Remaining Digits We have used the digit 5, leaving us with the remaining five digits: **{1, 2, 3, 4, 6}**. These five digits must be arranged in the first five empty positions.

Calculate the Number of Arrangements The number of ways to arrange 5 distinct items is 5 factorial (written as 5!).

- $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

Therefore, there are **120** different ways to arrange the remaining digits, resulting in 120 distinct six-digit numbers that are multiples of 5. 🔢

Problem 12:

To solve this, we need to find two numbers that meet two conditions: their product is 36 and their sum is 15.

1. **List Pairs of Numbers that Multiply to 36** First, let's list all the pairs of whole numbers whose product is 36.

   1 and 36

   2 and 18

   3 and 12

   4 and 9

   6 and 6

2. **Find the Pair that Adds Up to 15** Next, check which of these pairs has a sum of 15.

   $1 + 36 = 37$

   $2 + 18 = 20$

   $3 + 12 = 15$ (This is the correct pair!)

   $4 + 9 = 13$

   $6 + 6 = 12$

3. **Determine Sony's Age** The two ages are **3** and **12**. The problem states that Sophie is older than Sony.

   Sophie's age = 12

   Sony's age = 3

So, Sony is **3** years old. 🎂

Problem 13:

We can solve this problem by setting up and simplifying an algebraic equation.

Represent the Number Algebraically Let the four-digit number be represented by its digits abcd. Its value is 1000a + 100b + 10c + d. The reversed number, dcba, has a value of 1000d + 100c + 10b + a.

The first digit **a** and the last digit **d** cannot be 0, because both the original and the reversed numbers must be four-digit numbers.

Set Up and Simplify the Equation The problem states that the reversed number minus the original number is 4725.

- (1000d + 100c + 10b + a) - (1000a + 100b + 10c + d) = 4725
- 999d + 90c - 90b - 999a = 4725
- 999(d - a) + 90(c - b) = 4725

We can simplify this by dividing the entire equation by 9 (which is the greatest common divisor of 999, 90, and 4725).

- 111(d - a) + 10(c - b) = 525

Find the Difference Between the Digits

- In the equation 111(d - a) + 10(c - b) = 525, the term 10(c - b) must end in a 0. This means that the term 111(d - a) must end in a 5 for the sum to be 525.
- For 111 × (d - a) to end in 5, the value of **(d - a) must be 5**.
- Now we can solve for (c - b): 111(5) + 10(c - b) = 525 555 + 10(c - b) = 525 10(c - b) = -30 **(c - b) = -3**, which is the same as **(b - c) = 3**.

Count the Possible Digit Pairs Now we just need to count how many pairs of digits satisfy these two conditions.

- **Condition 1: d - a = 5** (where a and d are between 1 and 9) The pairs (a, d) are: (1, 6), (2, 7), (3, 8), and (4, 9). That's **4** possible pairs.
- **Condition 2: b - c = 3** (where b and c are between 0 and 9) The pairs (b, c) are: (3, 0), (4, 1), (5, 2), (6, 3), (7, 4), (8, 5), and (9, 6). That's **7** possible pairs.

Calculate the Total Number of Integers Since the choice for the first and last digits is independent of the choice for the middle two digits, we multiply the possibilities together.

- 4 (choices for a, d) × 7 (choices for b, c) = 28 🔢

Problem 14:

Let's use **C** for the number of correct answers and **I** for the number of incorrect answers. We can solve this by setting up a couple of simple equations based on the information given.

1. **Set Up the Equations**

The scoring gives us our first equation: C + (0.5 * I) = 25

The new piece of information gives us our second equation: C = 2 * I

2. **Substitute and Solve** Now we can substitute the second equation into the first one. Replace **C** with **2 * I** in the scoring equation:

(2 * I) + (0.5 * I) = 25

Combine the terms:

2.5 * I = 25

Finally, solve for **I**:

I = 25 / 2.5

I = 10

To double-check, if he had **10** incorrect answers, he must have had 2 * 10 = 20 correct answers. His score would be 20 + (0.5 * 10) = 20 + 5 = 25, which is correct. ✅

Problem 15:

To solve this, we'll use the divisibility rule for 3, which states that a number is a multiple of 3 if the sum of its digits is also a multiple of 3.

Analyze the First Number: 74A52B1 The sum of its digits must be a multiple of 3.

Sum 1 = 7 + 4 + A + 5 + 2 + B + 1 = 19 + A + B

For **19 + A + B** to be a multiple of 3, the value of **1 + A + B** must be a multiple of 3 (since 18 is a multiple of 3).

Analyze the Second Number: 326AB4C The sum of its digits must also be a multiple of 3.

Sum 2 = 3 + 2 + 6 + A + B + 4 + C = 15 + A + B + C

Since **15** is already a multiple of 3, the remaining part, **A + B + C**, must also be a multiple of 3.

Find the Condition for C Since both **Sum 1** and **Sum 2** are multiples of 3, their difference must also be a multiple of 3.

(Sum 2) - (Sum 1) = (15 + A + B + C) - (19 + A + B)

The A + B terms cancel out, leaving: 15 + C - 19 = C - 4

Therefore, **C - 4** must be a multiple of 3.

Find the Possible Values for C We need to find single digits (0-9) for C that make C - 4 a multiple of 3.

If C = 1, then 1 - 4 = -3 (This works).

If C = 4, then 4 - 4 = 0 (This works).

If C = 7, then 7 - 4 = 3 (This works).

The possible values for C are **1, 4, and 7**.

Determine the Largest Value The question asks for the **largest possible value** of C. From the set of possibilities {1, 4, 7}, the largest is **7**. ✅