ARCHIMEDES MATH COMPETITION

Additional Late Round 2024

Solution:

Problem 1:

Let the three positive whole numbers be x, y, and z. According to the problem, their sum is 47, so we can write the equation $x+y+z=47$. We are given that one of the numbers is 11, so let's set $x=11$. Substituting this into the equation gives us $11+y+z=47$. Subtracting 11 from both sides, we get $y+z=36$. We are also told that the second number is half of the third number, which can be written as $y=2z$, or equivalently, $z=2y$. Now we can substitute $z=2y$ into the equation $y+z=36$. This gives us $y+2y=36$. Combining the terms on the left side, we get $3y=36$. Dividing both sides by 3, we find that $y=12$. Now that we have the value of y, we can find the value of z using the equation $z=2y$. $z=2\times12=24$. So, the three numbers are 11, 12, and 24. The problem asks for the positive difference between the smallest and largest numbers. The smallest number is 11 and the largest number is 24. The difference is $24-11=13$.

The correct answer is 13.

Problem 2:

The pattern of digits {1, 2, 3, 4, 5, 6, 7, 8, 9, 0} repeats every 10 digits. To find the digit at a specific position, we can look at the remainder when the position number is divided by 10. The 31st digit in the pattern can be found by calculating $31\div10$, which is 3 with a remainder of 1. This means the 31st digit is the same as the 1st digit in the pattern, which is 1. The 32nd digit in the pattern can be found by calculating $32\div10$, which is 3 with a remainder of 2. This means the 32nd digit is the same as the 2nd digit in the pattern, which is 2. The 33rd digit in the pattern can be found by calculating $33\div10$, which is 3 with a remainder of 3. This means the 33rd digit is the same as the 3rd digit in the pattern, which is 3. The 34th digit in the pattern can be found by calculating 34÷10, which is 3 with a remainder of 4. This means the 34th digit is the same as the 4th digit in the pattern, which is 4. The 35th digit in the pattern can be found by calculating 35÷10, which is 3 with a remainder of 5. This means the 35th digit is the same as the 5th digit in the pattern, which is 5. The five digits are 1, 2, 3, 4, and 5. To find the sum, we add these digits together: $1+2+3+4+5=15$.

The correct answer is 15.

Problem 3:

From Peter's perspective, we can determine the total number of children in the family. Since Peter has 5 brothers, there are a total of 5+1 (Peter himself) =6 boys. Since Peter has 3 sisters, there are 3 girls. Now, let's look from his sister Joanna's perspective. Joanna is one of the 3 girls. The number of sisters Joanna has, denoted by S, is the total number of girls minus herself. So, $S=3-1=2$. The number of brothers Joanna has, denoted by B, is the total number of boys in the family. So, $B=6$. The problem asks for the product of S and B. The product is $S\times B=2\times 6=12$.

The correct answer is 12.

Problem 4:

The sum of the two numbers is **5**.

Let's label the grid cells by C(row, column). Based on the image, the initial grid is:

- C(1,1) = 1
- C(2,2) = 2
- C(2,3) = 4
- C(3,1) = B
- C(3,3) = 3
- C(4,3) = A

The goal is to find the values of **A** and **B** and then calculate their sum.

1. **Analyze the Main Diagonal** The main diagonal runs from top-left to bottom-right and contains C(1,1), C(2,2), C(3,3), and C(4,4). The

values we know are **1**, **2**, and **3**. Since each number from 1 to 4 must appear exactly once, the last cell, C(4,4), must be **4**.

2. **Analyze Column 3** This column contains C(1,3), C(2,3)=4, C(3,3)=3, and C(4,3)=A. The missing numbers are **1** and **2**. Therefore, the set of values for A and C(1,3) is {1, 2}.
3. **Analyze Row 1** This row starts with C(1,1)=1. This means that no other cell in this row can be 1. Therefore, C(1,3) cannot be 1. Looking back at our finding from Column 3, if C(1,3) can't be 1, it must be **2**.

- This forces the value of **A** to be **1** (since {A, C(1,3)} = {1, 2}).

So, **A = 1**. 👍

1. **Find the value of B**

Row 2: We have C(2,2)=2 and C(2,3)=4. The missing numbers are **1** and **3**. So, {C(2,1), C(2,4)} = {1, 3}.

Column 1: We have C(1,1)=1. We also know from the previous step that C(2,1) must be 3 (since it can't be 1).

Now Column 1 has C(1,1)=1 and C(2,1)=3. The missing numbers are **2** and **4**. The remaining cells are B and C(4,1). So, {B, C(4,1)} = {2, 4}.

Row 4: We have C(4,3)=1 and C(4,4)=4. The missing numbers are **2** and **3**. So, {C(4,1), C(4,2)} = {2, 3}.

By comparing the possibilities for C(4,1) from Column 1 ({2, 4}) and Row 4 ({2, 3}), we can see the only common value is **2**. So, C(4,1) = 2.

Since {B, C(4,1)} = {2, 4}, and we've determined C(4,1) = 2, then **B** must be **4**.

So, **B = 4**. 🎉

We have found that **A = 1** and **B = 4**.

The sum is A + B = 1 + 4 = 5.

1	4	2	3
3	2	4	1
4	1	3	2
2	3	1	4

Problem 5:

There are **33** integers remaining in the set.

The problem is equivalent to finding how many numbers from 1 to 100 are not divisible by 2 and not divisible by 3. We can find this by first calculating how many numbers are divisible by 2 or 3 and subtracting that from the total.

To find the total numbers removed, we use the Principle of Inclusion-Exclusion:

- First, we find the number of multiples of 2:

$$N_{mult.of2} = \left\lfloor \frac{100}{2} \right\rfloor = 50$$

- Next, we find the number of multiples of 3:

$$N_{mult.of3} = \left\lfloor \frac{100}{3} \right\rfloor = 33$$

- Adding these together double-counts the numbers divisible by both, which are the multiples of 6. We must subtract this overlap:

$$N_{mult.of6} = \left\lfloor \frac{100}{6} \right\rfloor = 16$$

- The total number of integers to be removed is:
  (Multiples of 2)+(Multiples of 3)−(Multiples of 6)=50+33−16=67

Finally, we subtract the total removed from the original number of integers in the set: 100−67=33

The correct answer is 33.

Problem 6:

Angles on a Straight Line: The problem states that PQR is a **straight line segment**. The sum of all angles on a straight line is always **180°**. From the figure, the sum of the angles at point Q is: $x^\circ+x^\circ+y^\circ+y^\circ+x^\circ=180^\circ$

Formulate an Equation: By combining the like terms, we get our first equation: $3x^\circ+2y^\circ=180^\circ$

Use the Given Information: The problem also gives us a second equation: $x^\circ+y^\circ=75^\circ$

Solve the System of Equations: We now have a system of two equations:

- $3x+2y=180$
- $x+y=75$

To solve for **x**, we can first rearrange the second equation to find y in terms of x: $y=75-x$

Now, substitute this expression for y into the first equation: $3x+2(75-x)=180$

Distribute the 2: $3x+150-2x=180$

Combine the x terms: $x+150=180$

Finally, subtract 150 from both sides to find **x**: $x=180-150$ $x=30$

Therefore, the value of **x** is **30**.

Problem 7:

Here's the logical process to find the number:

Set up the equation. Let the natural number be represented by N and its last digit by d. The problem states that the number is 5 times its last digit. This can be written as the equation: $N=5\times d$

Analyze the properties. Since N is 5 times an integer (d), N must be a multiple of 5. Any natural number that is a multiple of 5 must end in either 0 or 5. This means the last digit, d, can only be **0** or **5**.

Test the possibilities.

Case 1: The last digit is 0. If we set d=0, the equation gives us $N=5\times 0=0$. However, 0 is not a natural number (natural numbers are {1, 2, 3, ...}), so this case is not a valid solution.

Case 2: The last digit is 5. If we set $d=5$, the equation gives us $N=5\times5=25$. Let's check if this works. The number is 25, and its last digit is indeed 5. The condition is met.

Thus, **25** is the only natural number that satisfies the condition. None of the multiple-choice options provided are correct.

Problem 8:

Here's the breakdown of how to find the percentage of right-handed students. The easiest way is to first find the percentage of left-handed students and then subtract that from the total.

1. **Find the percentage of left-handed boys.**

60% of the students are boys.

15% of these boys are left-handed.

To find what percentage of the *entire class* this represents, we multiply the percentages: $0.15\times60\%=9\%$

So, **9%** of the total students are left-handed boys.

2. **Find the percentage of left-handed girls.**

If 60% of the students are boys, then 40% are girls ($100\%-60\%=40\%$).

10% of these girls are left-handed.

Multiply the percentages to find their portion of the whole class: $0.10\times40\%=4\%$

So, **4%** of the total students are left-handed girls.

3. **Find the total percentage of left-handed students.**

Add the percentages of left-handed boys and left-handed girls: $9\%+4\%=13\%$

In total, **13%** of the students in the class are left-handed.

4. **Find the percentage of right-handed students.**

Subtract the percentage of left-handed students from 100%: $100\%-13\%=87\%$

The correct answer is **87%**.

Problem 9:

You can create a total of **39** integers.

To find the total, we'll calculate the number of possibilities for each case (1-digit, 2-digit, and 3-digit) and then add them together. Since repetition is allowed, we have 3 choices (1, 2, or 3) for each position in the number.

1-Digit Integers You can form three 1-digit integers: 1, 2, and 3.

Number of possibilities = **3**

2-Digit Integers For a 2-digit number, there are two positions.

Choices for the first digit: 3 (1, 2, or 3)

Choices for the second digit: 3 (1, 2, or 3)

Number of possibilities = $3 \times 3 = 9$

3-Digit Integers For a 3-digit number, there are three positions.

Choices for the first digit: 3

Choices for the second digit: 3

Choices for the third digit: 3

Number of possibilities = $3 \times 3 \times 3 = 27$

Finally, add the possibilities from all three cases to get the **total**.

- Total = (1-digit numbers) + (2-digit numbers) + (3-digit numbers)
- Total = $3+9+27=39$

Problem 10:

The average age of the four daughters will be **34** years old.

The key to solving this problem is to use the constant **age differences** between the sisters to find their ages in the future.

1. **Find the Age Differences** First, let's determine how much older the other sisters are compared to Danielle, based on the initial information (Anna=11, Brianna=9, Carly=8, Danielle=4).

Anna is 11 - 4 = 7 years older than Danielle.

Brianna is 9 - 4 = 5 years older than Danielle.

Carly is 8 - 4 = 4 years older than Danielle.

2. **Calculate Their Ages When Danielle is 30** Now we can find the age of each sister when Danielle turns 30.

Danielle's age = **30**

Anna's age = 30 + 7 = 37

Brianna's age = 30 + 5 = 35

Carly's age = 30 + 4 = 34

3. **Calculate the Average Age** To find the average, add their ages together and divide by the number of daughters (4).

Sum of ages = 30 + 37 + 35 + 34 = 136

Average age = 136 / 4 = 34

When Danielle turned 30, the average age of the four sisters was **34**. 🎂

Problem 11:

To find the missing digits, we just need to perform the multiplication of 692 × 354 step-by-step.

1. **First Partial Product:** This line shows 692 × 4.
   - 692 × 4 = 2768. The erased digit here is **2**.
2. **Second Partial Product:** This line shows 692 × 5, shifted one place to the left.
   - 692 × 5 = 3460. The erased digits here are **3** and **6**.
3. **Third Partial Product:** This line shows 692 × 3, shifted two places to the left.
   - 692 × 3 = 2076. The erased digits here are **2** and **7**.
4. **Final Sum:** The last line is the sum of the partial products. By adding them up, we find the complete result.

   2768

   3460

   +2076

   244968

   - The full sum is 244,968. The erased digit in the final answer is **4**.
5. **Sum of Erased Digits:** Finally, add up all the digits you found.
   - 2+3+6+2+7+4=24 ✅

Problem 12:

The problem states that each number after the fourth is the **sum of the four numbers before it**. Let's look at the sequence:

3, A, B, C, 30, 57, ...

The sixth number in the sequence is **57**. According to the rule, 57 must be the sum of the four numbers that come before it (A, B, C, and 30). This gives us a simple equation:

A + B + C + 30 = 57

To find the value of A + B + C, you just need to subtract 30 from both sides of the equation.

A + B + C = 57 - 30 A + B + C = 27 ➕

Problem 13:

To find the total, you need to count how many digits are used for the 1-digit numbers, 2-digit numbers, and 3-digit numbers separately and then add them together.

1-Digit Numbers (1-9) There are **9** numbers from 1 to 9. Each has one digit.

- 9 numbers × 1 digit = 9 digits

2-Digit Numbers (10-99) There are **90** numbers from 10 to 99. Each has two digits.

- 90 numbers × 2 digits = 180 digits

3-Digit Number (100) There is **1** number, 100, which has three digits.

- 1 number × 3 digits = 3 digits

Total Digits Finally, add the digit counts from all the groups together.

- 9 + 180 + 3 = 192 digits ✍️

Problem 14:

An Alternative Method (Listing and Checking)

We can find the number by testing candidates that meet the conditions one by one.

List numbers that meet the first condition. A number that gives a remainder of **2** when divided by 5 must end in either a 2 or a 7.

Possible numbers: 7, 12, 17, 22, 27, 32, 37, 42, 47, ...

Filter that list using the second condition. Now, from the list above, let's find the numbers that give a remainder of **1** when divided by 6.

Is it 7? 7 ÷ 6 = 1 with a remainder of 1. **Yes.**

Is it 12? 12 ÷ 6 = 2 with a remainder of 0. No.

Is it 17? 17 ÷ 6 = 2 with a remainder of 5. No.

...continuing this pattern, the next number that works is 37.

Is it 37? 37 ÷ 6 = 6 with a remainder of 1. **Yes.**

The numbers that work for the first two conditions are **7, 37, 67, 97,** etc. (increasing by 30 each time).

Check the final condition. For our list of candidates (7, 37, 67, 97,...), we now check if the difference between their division quotients is **3**.

For the number 37:

37 ÷ 5 = 7 R 2 (The first quotient is 7)

37 ÷ 6 = 6 R 1 (The second quotient is 6)

The difference is 7 - 6 = 1. Not our number.

For the number 67:

67 ÷ 5 = 13 R 2 (The first quotient is 13)

67 ÷ 6 = 11 R 1 (The second quotient is 11)

The difference is 13 - 11 = 2. Not our number.

For the number 97:

97 ÷ 5 = 19 R 2 (The first quotient is 19)

97 ÷ 6 = 16 R 1 (The second quotient is 16)

The difference is 19 - 16 = 3. **This is the answer!** 👍

Problem 15:

To solve this, we can systematically find all possible combinations by fixing the first digit (hundreds place) and then finding the pairs for the other two digits. Remember, the first digit of a three-digit number can't be zero.

First Digit is 1: If the first digit is **1**, the other two digits must sum to 5 (6 - 1 = 5). The combinations are (0,5), (1,4), (2,3), (3,2), (4,1), and (5,0). That's **6** numbers (105, 114, 123, 132, 141, 150).

First Digit is 2: If the first digit is **2**, the other two digits must sum to 4 (6 - 2 = 4). The combinations are (0,4), (1,3), (2,2), (3,1), and (4,0).

That's **5** numbers (204, 213, 222, 231, 240).

First Digit is 3: If the first digit is **3**, the other two digits must sum to 3 (6 - 3 = 3). The combinations are (0,3), (1,2), (2,1), and (3,0).

That's **4** numbers (303, 312, 321, 330).**First Digit is 4:** If the first digit is **4**, the other two digits must sum to 2 (6 - 4 = 2). The combinations are (0,2), (1,1), and (2,0).

That's **3** numbers (402, 411, 420).

First Digit is 5: If the first digit is **5**, the other two digits must sum to 1 (6 - 5 = 1). The combinations are (0,1) and (1,0).

That's **2** numbers (501, 510).

First Digit is 6: If the first digit is **6**, the other two digits must sum to 0 (6 - 6 = 0). The only combination is (0,0).

That's **1** number (600).

Adding up all the possibilities gives the total count: 6+5+4+3+2+1=21 🔢