ARCHIMEDES MATH COMPETITION

Additional Late Round 2024

Solution:

Problem 1:

The sequence is 1, 2, 4, __, 11, 16.

Let's look at the differences between consecutive numbers:

- From 1 to 2, the difference is 2−1=1.
- From 2 to 4, the difference is 4−2=2.
- From 11 to 16, the difference is 16−11=5.

The pattern of differences seems to be increasing by 1: 1, 2, __, __, 5. This suggests the differences are 1, 2, 3, 4, 5.

Let's use this to find the missing number:

- The difference after 4 should be 3. So, 4+3=7.
- Let's check the next difference: The difference after 7 should be 4. So, 11−7=4. This matches the pattern.

Thus, the missing number is 7.

The final answer is C) 7.

Problem 2:

Find Bob's score: Bob scored 35 points.

Find Lilia's score: Lilia scored 15 points fewer than Bob.

Lilia's score = Bob's score - 15 Lilia's score = 35−15=20 points.

Find Carl's score: Carl scored twice as many points as Lilia.

Carl's score = 2 × Lilia's score Carl's score = 2×20=40 points.

Therefore, Carl scored 40 points.

The final answer is C) 40.

Problem 3:

To share the jelly beans equally, you just need to divide the total number of jelly beans by the number of friends.

Total jelly beans = 18 Number of friends = 3
Jelly beans per friend = 18÷3=6
Each friend should get 6 jelly beans.
The final answer is **E) 6**.

Problem 4:

Let the number of mice Kitty caught on the first day be x. According to the problem:

- On Day 1, Kitty caught x mice.
- On Day 2, Kitty caught x+2 mice.
- On Day 3, Kitty caught x+4 mice.

The total number of mice caught in these three days is the sum of mice caught each day: Total mice = x+(x+2)+(x+4) Total mice = 3x+6
We can factor out 3 from the total: Total mice = 3(x+2)
Since x represents the number of mice caught on Day 1, x must be a whole number (non-negative integer). Typically, "number of mice caught" implies x≥0. If x=0, total mice = 3(0+2)=6. If x=1, total mice = 3(1+2)=9. If x=2, total mice = 3(2+2)=12. And so on.
From the expression 3(x+2), we can see that the total number of mice caught must be a multiple of 3.
Now, let's check the given answer options to see which one is a multiple of 3: A) 14 (14 is not divisible by 3) B) 13 (13 is not divisible by 3) C) 8 (8 is not divisible by 3) D) 10 (10 is not divisible by 3) E) 9 (9 is divisible by 3)
The only option that is a multiple of 3 is 9. If Kitty caught 9 mice in total, then: 3(x+2)=9 x+2=3 x=1
This means Kitty caught 1 mouse on Day 1, 3 mice on Day 2, and 5 mice on Day 3. The sum is 1+3+5=9, which is consistent.
The final answer is **E) 9**.

Problem 5:

To find the number in the shaded box with letter C, we need to identify the pattern in the pyramid. In this type of pyramid puzzle, a common

pattern is that the value in a box is the sum of the values in the two boxes directly below it.

Let's apply this rule step-by-step from the bottom row upwards:

Bottom Row (given): 4, 3, 2, 1

Second Row from the bottom:

- The leftmost box (which has 7 already) should be 4+3=7. (This matches the given 7).
- The middle-left box (labeled A) should be 3+2=5. So, **A = 5**.
- The middle-right box (which has 3 already) should be 2+1=3. (This matches the given 3).
- So, the second row from the bottom is: **7, 5, 3**.

Third Row from the bottom:

- The leftmost box (labeled B) should be the sum of the two boxes below it in the second row: 7+5=12. So, **B = 12**.
- The rightmost box (which has 8 already) should be the sum of the two boxes below it in the second row: 5+3=8. (This matches the given 8).
- So, the third row from the bottom is: **12, 8**.

Top Row (Shaded Box C):

- The shaded box (C) should be the sum of the two boxes below it in the third row: 12+8=20. So, **C = 20**.

The number that should be placed in the shaded box with letter C is 20.

The final answer is **C) 20**.

Problem 6:

Let the unknown number be x.

Follow the steps described in the problem to build an equation:

1. **"A number is added to 3."** x+3
2. **"The sum is then multiplied by 5."** 5×(x+3) or 5(x+3)
3. **"When 8 is subtracted from the product, the result is 12."** 5(x+3)−8=12

Now, solve the equation for x:

- **Add 8 to both sides:** 5(x+3)=12+8 5(x+3)=20

- **Divide both sides by 5:** x+3=520 x+3=4
- **Subtract 3 from both sides:** x=4−3 x=1

The number is 1.

Let's check the answer:

1. Add 1 to 3: 1+3=4
2. Multiply the sum by 5: 4×5=20
3. Subtract 8 from the product: 20−8=12 The result is 12, which matches the problem statement.

The final answer is **A) 1.**

Problem 7:

We look for two numbers that multiply to 100 and add up to 29. Let's list pairs of factors of 100 and their sums:

- 1 and 100 (Sum = 101)
- 2 and 50 (Sum = 52)
- 4 and 25 (Sum = 29) - This is our pair!
- 5 and 20 (Sum = 25)
- 10 and 10 (Sum = 20)

The two numbers are 4 and 25. Their difference is 25−4=21.

The final answer is **D) 21.**

Problem 8:

1. **Represent the five consecutive whole numbers:** Since there are 5 numbers (an odd number), let the middle number be x. The five consecutive whole numbers can then be written as: x−2, x−1, x, x+1, x+2
2. **Set up the equation for their sum:** The sum of these five numbers is 20. (x−2)+(x−1)+x+(x+1)+(x+2)=20
3. **Solve for x:** Combine the x terms: x+x+x+x+x=5x Combine the constant terms: −2−1+0+1+2=0 So, the equation simplifies to: 5x=20 Divide by 5: x=20:5, x=4
4. **Identify the five numbers:** Substitute x=4 back into our representations:

- First number: $x-2=4-2=2$
- Second number: $x-1=4-1=3$
- Third number: $x=4$
- Fourth number: $x+1=4+1=5$
- Fifth number: $x+2=4+2=6$ The five consecutive whole numbers are 2, 3, 4, 5, and 6. (Check their sum: $2+3+4+5+6=20$, which is correct.)

5. **Calculate their product:** Product $= 2\times3\times4\times5\times6$ Product $= 6\times4\times5\times6$ Product $= 24\times30$ Product $= 720$

The final answer is **E) 720**.

Problem 9:

The number is 638*977, where * is a single digit.

1. **Try * = 0:** The number is 6,380,977. $6,380,977\div7=911,568$ with a remainder of 1. (Not divisible)
2. **Try * = 1:** The number is 6,381,977. $6,381,977\div7=911,711$. (Divisible by 7!)
3. **Try * = 2:** The number is 6,382,977. $6,382,977\div7=911,853$ with a remainder of 6. (Not divisible)
4. **Try * = 3:** The number is 6,383,977. $6,383,977\div7=911,996$ with a remainder of 5. (Not divisible)
5. **Try * = 4:** The number is 6,384,977. $6,384,977\div7=912,139$ with a remainder of 4. (Not divisible)
6. **Try * = 5:** The number is 6,385,977. $6,385,977\div7=912,282$ with a remainder of 3. (Not divisible)
7. **Try * = 6:** The number is 6,386,977. $6,386,977\div7=912,425$ with a remainder of 2. (Not divisible)
8. **Try * = 7:** The number is 6,387,977. $6,387,977\div7=912,568$ with a remainder of 1. (Not divisible)
9. **Try * = 8:** The number is 6,388,977. $6,388,977\div7=912,711$. (Divisible by 7!)
10. **Try * = 9:** The number is 6,389,977. $6,389,977\div7=912,853$ with a remainder of 6. (Not divisible)

The values of * for which the number 638*977 is divisible by 7 are **1** and **8**.

The final answer is 1 and 8.

Problem 10:

To find the number covered by the star, let's assign a variable to each symbol and solve the equations step-by-step.

Let:

- Smiley face = S
- Heart = H
- Diamond = D
- Star = X (This is the number we want to find)

From the image, we have the following equations:

1. S + H = 3
2. H + H = 4
3. H + D = 5
4. S + D = X

Now, let's solve them:

Step 1: Find the value of the Heart (H). From equation (2): H + H = 4 2H = 4 H = 4 / 2 H = 2

Step 2: Find the value of the Smiley (S). Substitute the value of H = 2 into equation (1): S + H = 3 S + 2 = 3 S = 3 - 2 S = 1

Step 3: Find the value of the Diamond (D). Substitute the value of H = 2 into equation (3): H + D = 5 2 + D = 5 D = 5 - 2 D = 3

Step 4: Find the value of the Star (X). Substitute the values of S = 1 and D = 3 into equation (4): S + D = X 1 + 3 = X X = 4

So, the number covered by the star is 4.

The final answer is 4.

Problem 11:

Let's write down the sums explicitly:

- **Angelica's sum:** 1+2+⋯+10+11+12+⋯+20
- **Benjamin's sum:** 11+12+⋯+20+21+22+⋯+30

Chris's calculation is: (Benjamin's sum) - (Angelica's sum).
Notice that the numbers from 11 to 20 are present in *both* Angelica's sum and Benjamin's sum. When Chris subtracts, these common numbers will cancel each other out.
So, Chris's number = (numbers unique to Benjamin's sum) - (numbers unique to Angelica's sum) Chris's number = $(21+22+\cdots+30)-(1+2+\cdots+10)$
Now, let's pair up the numbers from these two parts: $(21-1)+(22-2)+(23-3)+\cdots+(30-10)$
Look at each pair:

- $21-1=20$
- $22-2=20$
- $23-3=20$
- ...
- $30-10=20$

How many such pairs are there? There are 10 pairs (from 1 to 10, or 21 to 30). So, Chris's number is simply 10 times the value 20.
Chris's number = $10\times20=200$.
The final answer is 200.

Problem 12:
Determine the number of 4-point and 5-point questions. Let x be the number of 4-point questions. Let y be the number of 5-point questions.
We have two pieces of information to form equations:

- **Total number of questions:** There are 20 questions. $x+y=20$ (Equation 1)
- **Total maximum points:** The maximum possible score is 84 points. $4x+5y=84$ (Equation 2)

Now, we solve this system of equations. From Equation 1, we can express x as $x=20-y$. Substitute this into Equation 2: $4(20-y)+5y=84$ $80-4y+5y=84$ $80+y=84$ $y=84-80$ $y=4$
So, there are 4 questions worth 5 points. Now find x using Equation 1: $x+4=20$ $x=20-4$ $x=16$

Thus, there are 16 questions worth 4 points and 4 questions worth 5 points. (Check: 16×4+4×5=64+20=84 points, which is correct).

2. Calculate Jack's score. Jack solved all the 4-point questions and half of the 5-point questions.

- Points from 4-point questions: 16 questions×4 points/question=64 points.
- Points from 5-point questions: Jack solved half of the 4 five-point questions, which is 4÷2=2 questions. 2 questions×5 points/question=10 points.
- **Jack's total score:** Jack's score = Points from 4-point questions + Points from 5-point questions Jack's score = 64+10=74 points.

Jack's score is 74.

The final answer is 74.