ARCHIMEDES MATH COMPETITION

Additional Late Round 2024

Grade 11

Problem №1.

What is the units digit of $2021^{2021} + 2022^{2022} + 2023^{2023} + 2024^{2024} =$

A) 6 B) 7 C) 8 D) 9 E) 0

Problem №2.

Consecutive numbers are numbers that follow each other in order. They have a difference of 1 between every two numbers.

In how many ways can 105 be written as the sum of two or more consecutive positive integers?

A) 5 B) 7 C) 9 D) 11 E) 13

Problem №3.

What is the difference between the largest possible three-digit positive integer with no repeated digits and the smallest possible three-digit positive integer with no repeated digits?

A) 864 B) 885 C) 867 D) 858 E) 880

Problem №4.

Loretta had an average score of 91% on her first five math tests. In order to improve, she now wants to bring her average test score up to at least a 96%.

If it is possible for her to get a 100% (but no greater) on every upcoming test, what is the **minimum** number of additional tests that Loretta needs to take to do this?

A) 3 B) 4 C) 5 D) 6 E) 7

Problem №5.

Mrs. Smithfield wrote the following set of integers on the board:

$$\{11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$$

How many subsets of the above set contain **at least one prime number**?

A) 960 B) 768 C) 512 D) 256 E) 128

Problem №6.

The positive solution of the quadratic equation $x^2 = x + 1$ is a value called **the Golden Ratio**:

$$x = \frac{1 + \sqrt{5}}{2}$$

When this value (i.e. the Golden Ratio) is raised to the 12th power, the result may be expressed in the form $\boldsymbol{A} + \boldsymbol{B}\sqrt{\boldsymbol{C}}$, where $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C} \in Z^+$.

What is the **value** of $\boldsymbol{A} + \boldsymbol{B} + \boldsymbol{C}$?

A) 232 B) 234 C) 236 D) 238 E) 240

Problem №7.

How many positive numbers less than 3000 and divisible by 5 can be formed using the digits 0, 1, 2, 3, 4, 5, 6 if no digits are repeated?

A) 162 B) 157 C) 155 D) 152 E) 147

Problem №8.

Each letter in the 3 × 4 table below represents a positive whole number. Three rules are followed in the placement of numbers:

- The sum of the numbers in each of the three rows is the same.
- The sum of the numbers in each of the four columns is the same.
- The sum of any row does **not** equal the sum of any column.

What is the **sum** of $\boldsymbol{A} + \boldsymbol{B} + \boldsymbol{C} + \boldsymbol{D} + \boldsymbol{E}$?

78	*A*	76	75
B	81	80	*C*
D	85	*E*	83

A) 378 B) 478 C) 198 D) 298 E) 308

Problem №9.

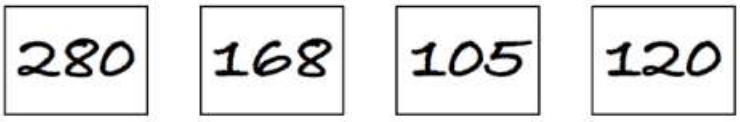
For the integers n and N, $1 < n < 1000$, $\log_3(\log_2 n) = N$.

What is the sum of the possible values of n?

Problem №10.

On the front of each of four cards, Jonah writes a positive integer, as shown, and then says that there is another positive integer hidden on the back of each of the four cards. To give some hints about the hidden numbers, Jonah says that the integer shown on each card is the product of the integers hidden behind the other three cards.

What is the **product** of all four hidden integers?



Problem 11.

Compute how many different integers from 100 to 400 inclusive are perfect powers (perfect squares, perfect cubes, etc.)

Problem №12.

ABC is an equilateral triangle. A circle with radius 1 is tangent to the line AB at the point B and to the line AC at point C. What is the side length of ABC?

Problem №13.

A two-meter long wire is cut into two parts so that the ratio of the length of the longer part to that of the shorter part equals the ratio of the length of the whole wire to that of the longer part. What is the length of the longer part ?

Problem №14.

The whole number N is 111 when written in base b, but it is 160 when written in base $b - 2$. What is N in base $b - 4$?

Problem №15.

There are 20 students in a class. If one new boy joins the class, there will be twice as many boys as girls in the class. What is the product of the number of boys and the number of girls in the class?