

# Additional Late Round 2024

### Grade 10

### Problem №1.

Find the next term of given sequence. 33, 35, 38, 43, 50, 61, 74, ...

- A) 95
- B) 94
- C) 93
- D) 92
- E) 91

# Problem №2.

How many positive perfect squares less than 2024 are divisible by 5?

- A) 8
- B) 9
- C) 10
- D) 11
- E) 12

### Problem No3.

If  $f(x) = \frac{x-1}{x+1}$  then which of the following is equal to f(x + 1) in terms of f(x)?  $(x \neq -1)$ 

A) 
$$-\frac{f(x)+1}{2f(x)}$$

$$\mathrm{B}) - \frac{f(x) + 1}{f(x)}$$

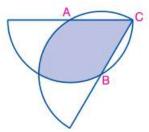
C) 
$$\frac{f(x)-1}{f(x)}$$

$$D) \frac{f(x)+1}{3-f(x)}$$

A) 
$$-\frac{f(x)+1}{2f(x)}$$
 B)  $-\frac{f(x)+1}{f(x)}$  C)  $\frac{f(x)-1}{f(x)}$  D)  $\frac{f(x)+1}{3-f(x)}$  E)  $-\frac{2f(x)}{f(x)+1}$ 

#### Problem №4.

 $\overline{AC}$  is the radius of the semicircle with center A, and  $\overline{BC}$  is the radius of the semicircle with center B. Point B is on the semicircle with center A, and point A is on the circle with center B. What is the area of the shaded region?



$$AC = BC = 12 cm$$

$$AC = BC = 12 \text{ cm}$$

- A)  $36\pi \ cm^2$  B)  $42\pi \ cm^2$  C)  $48\pi \ cm^2$  D)  $64\pi \ cm^2$  E)  $60\pi \ cm^2$

Problem №5.

 $a_n = \frac{n^2+1}{2n+1}$  is the general term of a sequence. If  $\frac{10}{3}$  is one of the terms of the sequence, then what is the next term?

A)  $\frac{65}{17}$  B)  $\frac{16}{9}$  C)  $\frac{26}{11}$  D)  $\frac{37}{13}$  E)  $\frac{122}{23}$ 

Problem №6.

Given for whole numbers x and y,  $(x + y) \cdot (2x - y) = 5$  then, what is the value of  $x \cdot y$ ?

A) 2 B) 6 C) 12 D) 15 E) 18

Problem №7.

Anna takes Math Test with two sections of 4 questions each. To pass the test, she needs to answer at least,

- 2 questions correctly in each section.
- 5 questions correctly in total.

Anna selects 5 out of these 8 questions randomly and answered all of them correctly. What is the probability that Anna passes this test?

A)  $\frac{3}{4}$  B)  $\frac{4}{5}$  C)  $\frac{5}{6}$  D)  $\frac{7}{8}$  E)  $\frac{6}{7}$ 

Problem №8.

A chessboard is hung up on the wall as a target, and three identical darts are thrown in its direction. In how many different ways can each dart hit the center of a different square such that any two of these three squares share at least one common vertex?

A) 345 B) 196 C) 64 D) 49 E) 4

Problem №9.

Suppose a, b are two numbers such that

$$a^2 + b^2 + 8a - 14b + 65 = 0.$$

Find the value of  $a^2 + ab + b^2$ .

#### Problem №10.

Using positive integers from 1 to n, sequence  $(a_n)$  is obtained by repeating each number squared times itself.

$$(a_n) = (1, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 3, 3, 3, 4, \dots, \underbrace{n, n, n, n, n, n, n, n, n}_{n^2 \text{ times}}, \dots)$$

What is the 205th term of  $(a_n)$ ?

#### Problem №11.

Let ABCD be a rectangle with AB = 30 and BC = 28. Point P and Q lie on  $\overline{BC}$  and  $\overline{CD}$  respectively so that all sides of  $\triangle ABP$ ,  $\triangle PCQ$ , and  $\triangle QDA$  have integer lengths. What is the perimeter of  $\triangle APQ$ ?

#### Problem №12.

An arithmetic sequence is a sequence in which each term after the first is obtained by adding a constant to the previous term. For example, 2, 5, 8, 11 and 1, 5, 9, 13 are arithmetic sequences.

In the grid shown, the numbers in each row must form an arithmetic sequence and the numbers in each column must also form an arithmetic sequence. What is the value of A+B+C+D?

5		11	Α
В	12		
9		C	
	D		

#### Problem №13.

The following two equations are known about the value of two numbers, a and b:

• Equation 1: a + b = 4

❖ Equation 2:  $a^2 + b^2 = 6$ 

What is the value of the expression  $a^3 + b^3$ ?

# Problem №14.

The sum of four positive integers is 150. When we increase the first number by 4, decrease the second number by 4, multiply the third by 4, and divide the fourth by 4, we find that the results are all the same. What is the **smallest** of the original four numbers?

### Problem №15.

The factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24. The sum of these factors, which write as S(24), is 1 + 2 + 3 + 4 + 6 + 8 + 12 + 24 = 60

If p is an odd prime number, what is the value of p given that  $S(2p^2) = 2613$ ?

arc-official.org Grade 10