Solution:

Problem 1:

The sum of the squares of the elements of S is **273**.

To find all possible values of x, we can analyze the system of equations by considering two cases: one where all variables are equal, and one where they are not.

If the variables are all equal, the three equations become identical. We can use the first one:

$x(x) + 4x = 60$

$x^2 + 4x - 60 = 0$

Factoring the quadratic: $(x + 10)(x - 6) = 0$ This gives two possible values for x: **-10** and **6**.

Let's subtract the second equation from the first one:

$(xy + 4z) - (yz + 4x) = 60 - 60$

$xy - yz + 4z - 4x = 0$

$y(x - z) - 4(x - z) = 0$

$(y - 4)(x - z) = 0$ This tells us that for any solution where the variables are not all equal, it must be true that either $y = 4$ or $x = z$.

If we substitute $y=4$ into the original system, the third equation becomes:

$zx + 4(4) = 60 \rightarrow zx = 44$ The first equation becomes:

$4x + 4z = 60 \rightarrow x + z = 15$ Now we have two numbers, x and z, whose sum is 15 and whose product is 44. These numbers must be **4** and **11**. So, x could be 4 or 11.

If x=z, the system reduces to two unique equations: $xy + 4x = 60$ and $x^2 + 4y = 60$. Solving this system yields the solutions we've already found: x=6 (which means y=z=6), x=-10 (which means y=z=-10), and x=4 (which gives y=11).

Combining all the solutions we found, the set S of possible values for x is **{-10, 4, 6, 11}**.

Finally, we find the sum of the squares of these values.

$(-10)^2 + 4^2 + 6^2 + 11^2$

$100 + 16 + 36 + 121 = 273$

The correct answer: 273

Problem 2:

There are **1,500** fish in the lake.

The problem states that the ratio of trout to the total number of fish is the same in the sample as it is in the entire lake. We can set this up as a proportion, where **T** is the total number of fish in the lake.

(Trout in Sample) / (Total Fish in Sample) = (Trout in Lake) / (Total Fish in Lake)

$30 / 180 = 250 / T$

First, simplify the fraction for the sample.

$30 / 180 = 1 / 6$ Now the equation is much simpler:

$1 / 6 = 250 / T$ By cross-multiplication, we find the value of T:

$1 \times T = 6 \times 250$, $T = 1500$

The correct answer: 1500

Problem 3:

One-third (**1/3**) of the interior of the large white circle is shaded.

The easiest way to solve this is to find the total area of the large circle and the total area of all the shaded parts, then find their ratio. We can use the grid to determine the dimensions. Let the side of a small grid square be 1 unit.

The diameter of the large circle spans 6 grid units, so its radius **$R = 3$**.

The total area is $\pi R^2 = \pi(3)^2 = 9\pi$.

The shaded area consists of two parts: the three small circles on the right and the larger, unusually shaped region on the left.

The diameter of each small circle is 1 unit, so their radius **$r_s = 0.5$**.

The area of one small circle is $\pi(0.5)^2 = 0.25\pi$.

The total area of the three small circles is $3 \times 0.25\pi = 0.75\pi$.

This shape is symmetrical about the horizontal axis. Let's find the area of its top half (in the top-left quadrant) and double it.The area of the top-left quadrant of the large circle is $(1/4) \times 9\pi = 2.25\pi$.This quadrant is partially filled by the top white semicircle. The radius of this medium circle is $3/2 = 1.5$, so the area of its semicircle is $(1/2) \times \pi(1.5)^2 = 1.125\pi$.

The shaded area in the top-left quadrant is the quadrant's area minus the white semicircle's area: $2.25\pi - 1.125\pi = 1.125\pi$.The bottom half is identical, so its shaded area is also 1.125π.

Total area of the large shaded region = $1.125\pi + 1.125\pi = 2.25\pi$.

Total shaded area = (large shaded region) + (three small circles) = 2.25π + 0.75π = 3π.

Fraction = (Total Shaded Area) / (Total Large Area)

Fraction = (3π) / (9π) = 1/3

The correct answer: 1/3

Problem 4:

There are **81** three-digit numbers that are divisible by 11.

The three-digit numbers are the integers from 100 to 999.

To find the first three-digit number divisible by 11, we can divide 100 by 11. 100 ÷ 11 = 9 with a remainder of 1. This means 11 × 9 = 99 is the last two-digit multiple. The first three-digit multiple is the next one: 99 + 11 = 110. So, the first number is **110** (= 11 × 10).

To find the last three-digit number divisible by 11, we can divide 999 by 11. 999 ÷ 11 = 90 with a remainder of 9. This means the largest multiple of 11 less than 999 is 999 - 9 = 990. So, the last number is **990** (= 11 × 90).

The multiples of 11 in this range are 11×10, 11×11, ..., 11×90. To find how many there are, we just need to count the integers from 10 to 90, inclusive.

Count = Last - First + 1

Count = 90 - 10 + 1 = 81

The correct answer: 81

Problem 5:

The sum of the first 6 terms is **728**.

Let the first term of the sequence be **a** and the common ratio be **r**. We can write the given information as two equations:

Equation 1: $a_6 - a_5 = ar^5 - ar^4 = 324$ **Equation 2:** $a_2 - a_1 = ar - a = 4$

First, let's factor both equations:

$ar^4(r - 1) = 324$

$a(r - 1) = 4$ Now, we can divide the first equation by the second equation. The a(r-1) terms will cancel out, leaving:

$r^4 = 324 / 4$, $r^4 = 81$, $r = 3$. Substitute $r = 3$ back into the second equation to find **a**:

$a(3 - 1) = 4$, $2a = 4$, $a = 2$

The first term is **2** and the common ratio is **3**. We can find the sum of the first 6 terms using the formula $S_n = a(r^n - 1) / (r - 1)$.

$S_6 = 2(3^6 - 1) / (3 - 1)$

$S_6 = 2(729 - 1) / 2$

$S_6 = 728$ The correct answer: 728

Problem 6:

There are **2** ways this can be done.

First, we need to find the sum of all the numbers on the cards. The sum of the integers from 1 to 9 is:

$1 + 2 + \ldots + 9 = 45$ Since these cards must be divided into 3 groups with an equal sum, each group must sum to: $45 / 3 = 15$

Next, we list all possible combinations of three different numbers from the set {1, 2, ..., 9} that add up to 15.

{1, 5, 9},{1, 6, 8},{2, 4, 9},{2, 5, 8},{2, 6, 7},{3, 4, 8},{3, 5, 7},{4, 5, 6}

From the list above, we need to find three groups that use all nine numbers from 1 to 9 without any overlap. A systematic search reveals that there are only two possible ways to partition the cards:

Group 1: {1, 5, 9},Group 2: {2, 6, 7},Group 3: {3, 4, 8}

Group 1: {1, 6, 8},Group 2: {2, 4, 9},Group 3: {3, 5, 7}

Since there are only two valid combinations of three groups, this can be done in **2** ways. The correct answer: 2

Problem 7:

The first term is **5**.

Let the first term be $\mathbf{a_1}$ and the second term be $\mathbf{a_2}$. According to the rule, each subsequent term is the product of the previous two.

$a_3 = a_1 \times a_2$, $a_4 = a_2 \times a_3 = a_2 \times (a_1a_2) = a_1a_2^2$, $a_5 = a_3 \times a_4 = (a_1a_2) \times (a_1a_2^2)$ $= a_1^2a_2^3$, $a_6 = a_4 \times a_5 = (a_1a_2^2) \times (a_1^2a_2^3) = a_1^3a_2^5$

We are given that the sixth term is 4000. $a_1^3a_2^5 = 4000$

To solve for the integers $\mathbf{a_1}$ and $\mathbf{a_2}$, let's find the prime factorization of 4000. $4000 = 4 \times 1000 = 4 \times 10^3 = 2^2 \times (2\times5)^3 = 2^2 \times 2^3 \times 5^3 = 2^5 \times 5^3$

Now we can compare the two forms of the equation: $a_1^3 \times a_2^5 = 5^3 \times 2^5$ By matching the bases to the exponents, we can see that:

a_1^3 must be 5^3, so $\mathbf{a_1 = 5}$. a_2^5 must be 2^5, so $\mathbf{a_2 = 2}$.

The first term of the sequence is **5**. The correct answer: 5

Problem 8:

The height h of ΔABC is **14.6** units.

When a line is drawn parallel to the base of a triangle, it creates a smaller triangle at the top that is similar to the original one. The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding heights. Let the total area of ΔABC be **A** and its total height be **h**.

In the first figure, the unshaded part is a smaller triangle with a height of **11**. Its area is $A * (11/h)^2$. The shaded area is the total area minus this unshaded part: $\text{Shaded Area}_1 = A - A * (11/h)^2 = A * (1 - 121/h^2)$

In the second figure, the shaded part is the smaller triangle at the top. Its height is the total height minus the height of the unshaded trapezoid: h - 5. The area of this shaded triangle is: $\text{Shaded Area}_2 = A * ((h - 5)/h)^2$

The problem states that the two shaded areas are equal.

$A * (1 - 121/h^2) = A * ((h - 5)/h)^2$ We can cancel **A** from both sides and multiply by $\mathbf{h^2}$ to clear the denominators. $h^2 - 121 = (h - 5)^2$

$h^2 - 121 = h^2 - 10h + 25$ The $\mathbf{h^2}$ terms cancel out:

$-121 = -10h + 25$, $10h = 121 + 25$, $10h = 146$, $h = 14.6$

The correct answer: 14.6