



Second Round 2022-2023

### Solution:

#### Problem 1:

Each volume has 440 pages.

Let  $n$  be the number of pages in each volume. Since the pages are numbered consecutively across all three volumes:

The fifth page of Volume 1 is page 5.

The page numbers in Volume 2 start right after Volume 1 ends (at page  $n$ ), so the fifth page of Volume 2 is  $n + 5$ .

The page numbers in Volume 3 start after Volume 2 ends (at page  $2n$ ), so the fifth page of Volume 3 is  $2n + 5$ .

The problem states that the sum of these three page numbers is 1335.

$$5 + (n + 5) + (2n + 5) = 1335$$

Now, we can simplify and solve the equation for  $n$ .

$$3n + 15 = 1335$$

$$3n = 1320$$

$$n = 1320 / 3$$

$$n = 440$$

There are 440 pages in each volume.

The correct answer: 440

#### Problem 2:

The largest possible value of A is 24.

The problem states that the sum of any four consecutive numbers is 41.

Let's write this out:

$$A + B + C + D = 41$$

$B + C + D + E = 41$  By comparing these first two sums, we can see that  $A = E$ . Following this logic for the rest of the sequence ( $B+C+D+E = C+D+E+F$ , etc.), we find that the sequence is periodic with a period of 4:

$$A = E$$

$$B = F$$

$$C = G$$

$$D = H$$

Now we can use this pattern to simplify the information from the problem.

The sum of a full cycle is  $A + B + C + D = 41$ .

The condition  $B + G = 16$  becomes  $B + C = 16$ , since  $C = G$ .

We can substitute the second equation into the first:

$$A + (B + C) + D = 41$$

$$A + 16 + D = 41$$

$$A + D = 25$$

To get the largest possible value for  $A$ , we need to make  $D$  as small as possible. Since all the numbers must be positive natural numbers, the smallest possible value for  $D$  is 1.

$$A + 1 = 25$$

$$A = 24$$

The correct answer: 24

### Problem 3:

The sum of the prime factors of 2010 is 77.

First, we need to break down the number 2010 into its prime factors.

Since 2010 ends in 0, it's divisible by 10:  $2010 = 201 \times 10$ .

$$10 = 2 \times 5.$$

The sum of the digits of 201 is  $2 + 0 + 1 = 3$ , so it's divisible by 3:  $201 = 3 \times 67$ .

The number 67 is a prime number.

The full prime factorization of 2010 is  $2 \times 3 \times 5 \times 67$ .

The distinct prime factors are 2, 3, 5, and 67. Now, we just add them together.

$$2 + 3 + 5 + 67 = 77 \quad \checkmark$$

The correct answer: 77

#### Problem 4:

Melinda's father was 32 years old when Melinda was born.

In 2012, the sum of the ages of the three people was 86. Six years later, in 2018, each of the three people will be 6 years older.

Total increase in age = 3 people  $\times$  6 years = 18 years.

Sum of ages in 2018 = 86 + 18 = 104 years.

In 2018, the ratio of their ages (Father : Mother : Melinda) was 6 : 5 : 2.

We can represent their ages as 6x, 5x, and 2x. The sum of their ages is 104.

$$6x + 5x + 2x = 104$$

$$13x = 104$$

$x = 104 / 13 = 8$  Now we can find their exact ages in 2018:

**Father's Age:**  $6x = 6 \times 8 = 48$

**Mother's Age:**  $5x = 5 \times 8 = 40$

**Melinda's Age:**  $2x = 2 \times 8 = 16$

The father's age when Melinda was born is simply the difference between their current ages.

$$48 \text{ (Father's age)} - 16 \text{ (Melinda's age)} = 32$$

The correct answer: 32

#### Problem 5:

The value of k is 6.

The **Factor Theorem** states that if  $(x - a)$  is a factor of a polynomial, then the polynomial will equal zero when you substitute  $x = a$ . In this case, the factor is  $x + 3$ , which means  $x = -3$ .

We substitute  $x = -3$  into the polynomial and set the entire expression equal to zero.

$$3x^3 + kx^2 - 7x + 6 = 0$$

$$3(-3)^3 + k(-3)^2 - 7(-3) + 6 = 0$$

Now, we simplify and solve the equation for k.

$$\begin{aligned}
3(-27) + k(9) + 21 + 6 &= 0 \\
-81 + 9k + 27 &= 0 \\
9k - 54 &= 0 \\
9k &= 54 \\
k &= 6
\end{aligned}$$

The correct answer: 6

### Problem 6:

Bob will spend **11,400** more seconds reading than Chandra.

First, let's find the difference in the time it takes Bob and Chandra to read a single page.

Bob's time per page: 45 seconds

Chandra's time per page: 30 seconds

Difference =  $45 - 30 = 15$  seconds per page.

To find the total difference over the entire book, we multiply this per-page difference by the total number of pages.

$15 \text{ seconds/page} \times 760 \text{ pages} = 11,400 \text{ seconds.}$

The correct answer: 11400

### Problem 7:

The largest possible product is **280**.

To get the largest possible **positive** product by multiplying three numbers, there are two main strategies to consider:

**Multiply the three largest positive numbers.**

**Multiply the two smallest (most negative) numbers and the largest positive number.** (Because  $\text{negative} \times \text{negative} = \text{positive}$ ).

Let's calculate the result for both cases.

The three largest positive numbers in the set are 8, 6, and 4.

$$8 \times 6 \times 4 = 192$$

The two smallest (most negative) numbers are -7 and -5, and the largest positive number is 8.

$$(-7) \times (-5) \times 8 = 35 \times 8 = 280$$

By comparing the two results, we can see that **280** is the largest possible product.

The correct answer: 280

### **Problem 8:**

There are **7** numbers in the set.

The mean (or average) of a set of numbers is calculated by dividing the sum of the numbers by the count of how many numbers are in the set.

Mean = (Sum of Numbers) / (Count of Numbers)

We can rearrange this formula to solve for the count of numbers:

Count of Numbers = (Sum of Numbers) / Mean Now, we just need to plug in the values given in the problem.

Count of Numbers =  $448 / 64 = 7$

The correct answer: 7