Second Round 2022-2023

Solution:

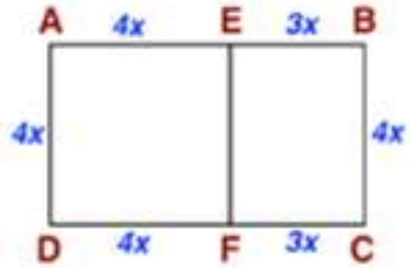
Problem 1:

A is at most 999, so A+32 is at most 1031. The minimum value of A+32 is 1000. However, the only palindrome between 1000 and 1032 is 1001, which means that A+32 must be 1001. It follows that A is 969, so the sum of the digits is 24.

Correct answer: 24

Problem 2:

Given the ratios of the sides, the sides may be represented by values shown.



As the perimeter of rectangle ABCD is 88 units, we have

$$88 = 22x => x = 4$$

Therefore, the area of rectangle ABCD is

$$A = (4x)(7x) = (4 \times 4) \times (7 \times 4) = (16) \times (28) = 448$$

Correct answer: 448

Problem 3:

We have $161 = 7 \times 23$. Given that A has two digits, the factors must be 23 and 7, which means that A is 32 and B is 7.Therefore, the correct product of the two numbers is $32 \times 7 = 224$

Correct answer: 224

Problem 4:

The sum of the two three-digit numbers is another three-digit number. Hence, as 1 carries over from adding 4 and 9, we get A+3=B. Since nothing is carried over from here, the maximum value of B is 9 when A=6. However, 593 is not divisible by 3. If B=8 then A=5, but once again, 583 is not divisible by 3. However, if B=7 then A=4 and 573 is divisible by 3. Hence, the largest possible value of A is 4.

Correct answer: 4

Problem 5:

A number that is divisible by both 5 and 11 must also be divisible by 55. Therefore, the numbers divisibly by 5 and 11 between 500 and 900 are 550, 605, 660, 715, 770, 825 and 880. Of these, the odd integers are 605, 715 and 825 and 605+715+825=2145.

Correct answer: 2145

Problem 6:

There are two options for buying tickets for 46 students:

Option A:

If we buy 4 group tickets plus 6 single tickets for the 46 students, we pay $4 \times 30 + 6 \times 6 = 120 + 36 = 156$ dollars.

Option B:

If we buy 5 group tickets, we pay $5 \times 30 = 150$ dollars, which is the cheaper option.

Correct answer: 150

Problem 7:

Every prime number other than 2 is odd. Since a, b and c are all prime numbers and a+b+c=22, which is even, it cannot be the case that all three prime numbers are odd (otherwise a+b+c would be odd).

Thus, one of the three prime numbers is 2, which means the sum of the other two prime numbers is 20. This can happen as 3+17 or 7+13, and no other way. Therefore, there are only two triples of primes that satisfy the requirements, and they are {2, 3, 17} and {2, 7, 13}.

Correct answer: 2

Problem 8:

There are only 5 students who have neither cat or dog, so 34-5=29. But 15+18=33 so there are 33-29=4 who have both cats and dogs. Since 18 have cats and 4 have both cats and dogs, then there are 18-4=14 who have cats only. This can be illustrated using a Venn diagram.

Correct answer: 14