ARCHIMEDES MATH COMPETITION

Second Round 2022-2023

Solution:

Problem 1:

The value of X is **47**.

The key to solving this puzzle is realizing that the sum of all the numbers in the grid can be found in two ways: by adding the row totals or by adding the column totals. The result must be the same either way.

First, let's find the sum of all numbers in the grid by adding the totals for each of the four columns.

30 + 53 + 27 + 40 = 150 The grand total of all numbers in the grid is **150**.

The grand total must also be equal to the sum of the three row totals.

(Row 1 Sum) + (Row 2 Sum) + (Row 3 Sum) = 150

37 + X + 66 = 150

Now, we can solve this simple equation to find the value of X.

103 + X = 150

X = 150 - 103

X = 47

Correct answer: 47

Problem 2:

The perimeter of the rectangle can be written as $2 \times [(3a - b) + (2a + 3b)]$, which may be simplified to $2 \times [5a + 2b]$. Given that $a = 1$, and that the perimeter is 18 cm, we have $18 = 2 \times [5 + 2b]$, which when solved gives $b = 2$. Therefore, the sides of the rectangle are 1 cm and 8 cm, which in turn gives the area of the rectangle as 8 cm^2.

Correct answer: 8

Problem 3:

If the integer is $\overline{AB} = 10A + B$. Thus, the reversed digit has a value of $10B + A$. The difference between these two then is $(10A + B) - (10B + A) = 9A - 9B$. As this is equal to 36, we have that

$$9A - 9B = 36 => A - B = 4.$$

Correct answer: 4

Problem 4:

The smallest two-digit number that can be repeated three times and result in a six-digit number greater than 500000 is 50 (resulting in 505050).

The largest two-digit number that can be repeated three times and result in a six-digit number smaller than 1000000 is 99 (resulting in 999999).

Thus, the desired six-digit numbers start with 50 (giving the number 5050050), 51 (giving the number 515151), 52 (giving the number 525252), …, 99 (giving the number 999999). Therefore, there are exactly 50 such six-digit numbers.

Correct answer: 50

Problem 5:

Oscar should choose the smallest possible three-digit number in order to get the greatest possible result. To check this answer, we can follow the process that Oscar goes through and note that choosing 100 does in fact provide the greatest possible final result:

- Number 719: $(3000 - 719) \times 3 = 2281 \times 3 = 6843$
- Number 222: $(3000 - 222) \times 3 = 2278 \times 3 = 8334$
- Number 100: $(3000 - 100) \times 3 = 2900 \times 3 = 8700$
- Number 451: $(3000 - 451) \times 3 = 2549 \times 3 = 7647$
- Number 989: $(3000 - 989) \times 3 = 2011 \times 3 = 6033$

Correct answer: 100

Problem 6:

The remainder is **4**.

A key rule in modular arithmetic is that the remainder of a product is the same as the product of the remainders. To find the remainder of $1492 \times 1776 \times 1812 \times 1996$ when divided by 5, we only need to find the remainder of each individual number, multiply them, and then find the final remainder.

The remainder of a number when divided by 5 is determined by its last digit.

The remainder of 149**2** divided by 5 is **2**.

The remainder of 177**6** divided by 5 is **1**.

The remainder of 181**2** divided by 5 is **2**.

The remainder of 199**6** divided by 5 is **1**.

Now, we multiply these remainders together.

$2 \times 1 \times 2 \times 1 = 4$

The remainder of this result (4) when divided by 5 is **4**.

Correct answer: 4

Problem 7:

An integer N divided by 6 with a remainder of 5 can be written as $N = 6x + 5$ where x is the quotient. Multiplying both sides of the equation by 9, we get $9N = 9(6x + 5) = 54x + 45$. Since 45 can be written as 42+3 then we get the equation $9N = 54x + 42 + 3 = (54x + 42) + 3$. Factoring out 6 gives $9N = 6(9x + 7) + 3$. Given that $9x + 7$ is an integer, then the remainder is 3.

Correct answer: 3

Problem 8:

There are **11** piano players.

We can find the number of piano players using the Principle of Inclusion-Exclusion.

The formula for two overlapping groups is: Total in Either Group = (Group 1) + (Group 2) - (Both Groups) In this case: Piano or Violin = (Piano Players) + (Violin Players) - (Both)

We can plug the numbers from the problem into this formula:

14 = (Piano Players) + 8 - 5

14 = (Piano Players) + 3

Piano Players = 14 - 3 = 11

Correct answer: 11