ARCHIMEDES MATH COMPETITION

Second Round 2022-2023

Solution:

Problem 1:

Going from right to left we have A=16-10=6, B=10-6=4, C=6-4=2, D=4-2, and E=2-2=0. Therefore, A+B+C+D+E=6+4+2+2+0=14

ANS: 14

Problem 2:

- The smaller number indicated by the arrow is 49.
- The larger number indicated by the arrow is 72.

Therefore, the difference between these two numbers is 72-49=23.

ANS: 23

Problem 3:

Let the missing numbers be represented by letters A through E, as shown.

A	9	B
3	C	D
8	E	6

Considering the leftmost column and the topmost row, we may fine the following:

$$A + 3 + 8 = A + 11 \Leftrightarrow A + 9 + B$$

$$11 = 9 + B \Rightarrow B = 2$$

Similarly, considering the two diagonals, we may find the following:

$$A + C + 6 \Leftrightarrow 8 + C + B = 8 + C + 2$$

$$A + 6 = 8 + 2 \Rightarrow A = 4$$

Given A=4 and B=2, we find that the "magic sum" must be 15, which in turn gives C=5, D=7 and E=1. Therefore, the sum of the five missing numbers is A+B+C+D+E=4+2+5+7+1=19

ANS: 19

Problem 4:

The fraction $\frac{31}{30}$ may be written as the sum of the unit fraction as shown:

$$\frac{31}{30} = \frac{1}{2} + \frac{1}{3} + \frac{1}{5}$$

Therefore, as A=2, B=3 and C=5, A+B+C=2+3=5=10

ANS: 10

Problem 5:

Percentage of girls is 100%-60%=40%. Percentage of left-handed students is

$\left(\frac{15}{100} \times 60\right) + \left(\frac{10}{100} \times 40\right) = \frac{900}{100} + \frac{400}{100} = 13\%$. Therefore, the percentage of right handed students in the class is 100%-13%=87%.

ANS: 87%

Problem 6:

As the square is an 8-by-8 square, its area is $8 \times 8 = 64$ square units. Given the areas of three of its regions, we find that the area of the fourth region (i. e. rectangle) must be

$$64 - (24 + 8 + 20) = 64 - 52 = 12.$$

ANS: 12

Problem 7:

The sum of these numbers can be grouped as allows:

$$(1 + 2 - 3 - 4) + (5 + 6 - 7 - 8) + (9 + 10 - 11 - 12) + \cdots + 301 + 302$$

The first 300 numbers listed give $300 \div 4 = 75$ groups of 4. The value of each of these grouped numbers is -4. Therefore, the sum/difference of the first 300 numbers is 300 numbers is $75\times (-4) = 300$. Adding the 301 and the 302 to this -300 gives 303.

ANS: 303

Problem 8:

The following must be noted about these special numbers:

- Given that the first digit must be even, it can be 2, 4, 6 or 8.
- As the two-digit number formed by the last two digits must be prime, it can be 23 or 29.
- There are no restrictions for the second digit, which means it may be 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9.

Overall, the number of possible numbers with the given properties is

$$4 \times 2 \times 10 = 80$$