Solution:

Problem 1:

The value of the expression is **2020**.

Understand the Conditions The problem states that **a** and **b** are non-zero additive inverses (opposites). This gives us two key pieces of information:

- $a + b = 0$
- $a / b = -1$ (since $a = -b$ and they are non-zero)

Simplify the Expression Now we can substitute these values into each part of the expression.

- **First Term:** $2(a + b - 1)^{2019}$

  Substitute $a + b = 0$: $2(0 - 1)^{2019} = 2(-1)^{2019}$

  Since 2019 is an odd number, $(-1)^{2019} = -1$.

  $2 \times (-1) = -2$
- **Second Term:** $3(a/b)^{2020}$

  Substitute $a/b = -1$: $3(-1)^{2020}$

  Since 2020 is an even number, $(-1)^{2020} = 1$.

  $3 \times 1 = 3$

Calculate the Final Value Finally, combine the simplified terms with the constant at the end.

- (First Term) + (Second Term) + 2019
- $-2 + 3 + 2019 = 1 + 2019 = 2020$ 💡

Problem 2:

The book has **189** pages.

We can find the total number of pages by figuring out how many pages of each length (1-digit, 2-digit, 3-digit) the book contains.

Count Digits for 1-Digit Pages (1-9)

- There are 9 single-digit pages.

- Digits used: 9 pages × 1 digit/page = 9 digits.
- Digits remaining: 459 - 9 = 450 digits.

Count Digits for 2-Digit Pages (10-99)

- There are 90 two-digit pages (from 10 to 99).
- Digits used: 90 pages × 2 digits/page = 180 digits.
- Digits remaining: 450 - 180 = 270 digits.

Count 3-Digit Pages (100 onwards) The remaining **270** digits are all used for 3-digit pages.

- Number of 3-digit pages = 270 digits / 3 digits/page = 90 pages.

Find the Total Number of Pages Finally, add up the number of pages from each category.

- 9 (1-digit) + 90 (2-digit) + 90 (3-digit) = 189 pages. 📖

Problem 3:

The value of the expression is **9.72**.

Factor the Expression The easiest way to approach this is to first simplify the expression $b^2 + bc - ab - ca$ by factoring it. We can do this by grouping the terms:

$(b^2 + bc) - (ab + ca)$

$b(b + c) - a(b + c)$ Now, we can factor out the common term $(b + c)$:

$(b + c)(b - a)$

Find the Value of (b - a) We are given the value of $(b + c)$, but we need to find the value of $(b - a)$. We can do this by subtracting the first given equation from the second one.

$(b + c) - (a + c) = 6.48 - 4.98$

$b - a = 1.5$

Calculate the Final Answer Now we can substitute the known values into our factored expression $(b + c)(b - a)$.

- $(6.48) \times (1.5) = 9.72$ 🧠

Problem 4:

The correct answer is **E) None of preceding**. The actual sum of the area is 119.

Find the Area of the Large Outer Square

- Let the length of one of the identical rectangles be **L** and the width be **w**. From the diagram, we can see that the side length of the large outer square is equal to L + w.
- We are given that the perimeter of each rectangle is 24 cm. The formula for the perimeter is 2(L + w).
- 2(L + w) = 24, which means L + w = 12.
- So, the side length of the large outer square is **12 cm**.
- The area of the large outer square is $12 \times 12 = 144$ cm^2.

Find the Area of the Center Square

- We are given that the perimeter of the center square is **20 cm**.
- The side length of the center square is 20 / 4 = 5 cm.
- The area of the center square is $5 \times 5 = 25$ cm^2.

Calculate the Area of the Border The sum of the areas of the four outer rectangles is the area of the large square minus the area of the center square.

- Area of Border = Area of Large Square - Area of Center Square
- Area of Border = 144 - 25 = 119 cm^2

Since 119 is not among options A, B, C, or D, the correct choice is **E) None of preceding**. 🖼️

Problem 5:

The value of the expression is **101/99**.

The key to solving this problem is to simplify the expression by factoring out the lowest common power of 10 from both the numerator and the denominator.

Factor the Numerator The lowest power of 10 in the numerator ($10^{2019} + 10^{2017}$) is 10^{2017}. We can factor this out:

- $10^{2017}\ (10^2 + 1)$
- $10^{2017}\ (100 + 1) = 10^{2017}\ (101)$

Factor the Denominator Similarly, we can factor 10^{2017} out of the denominator (10^{2019} - 10^{2017}):

- 10^{2017} (10^2 - 1)
- 10^{2017} (100 - 1) = 10^{2017} (99)

Simplify the Expression Now, we can write the full expression and cancel the common terms.

- [10^{2017} (101)] / [10^{2017} (99)]
- The 10^{2017} terms cancel out, leaving: **101 / 99** 💡

Problem 6:

There are **12** positive integer values of x.

Simplify the Expression First, we can simplify the given expression by dividing each term in the numerator by **x**:

- (x^2 - 7x + 60) / x = (x^2/x) - (7x/x) + (60/x)
- This simplifies to: x - 7 + 60/x

Analyze the Condition The problem states that **x** is a positive integer.

- This means that x - 7 will always be an integer.
- For the entire expression (x - 7) + 60/x to be an integer, the term **60/x** must also be an integer.

Find the Number of Values For 60/x to be an integer, **x** must be a positive divisor (or factor) of 60. So, the problem is now just a matter of counting the number of positive divisors of 60.

The divisors of 60 are: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, and 60.

Counting these numbers, we find there are a total of **12** divisors. 🧠

Problem 7:

There are **225** such numbers.

To solve this, we need to count the three-digit numbers that are **even** and have **exactly one odd digit**. We can break this down into cases based on the position of that single odd digit.

The digits we can use are:

- **Odd:** {1, 3, 5, 7, 9} (5 choices)
- **Even:** {0, 2, 4, 6, 8} (5 choices)

Case 1: The odd digit is in the hundreds place. The number must have the form **Odd-Even-Even**.

- **Hundreds digit:** 5 choices (1, 3, 5, 7, 9).
- **Tens digit:** 5 choices (0, 2, 4, 6, 8).
- **Ones digit:** 5 choices (0, 2, 4, 6, 8) to ensure the number is even.
- Total numbers for this case = $5 \times 5 \times 5 = 125$.

Case 2: The odd digit is in the tens place. The number must have the form **Even-Odd-Even**.

- **Hundreds digit:** 4 choices (2, 4, 6, 8, since it can't be 0).
- **Tens digit:** 5 choices (1, 3, 5, 7, 9).
- **Ones digit:** 5 choices (0, 2, 4, 6, 8) to ensure the number is even.
- Total numbers for this case = $4 \times 5 \times 5 = 100$.

Case 3: The odd digit is in the ones place. The number would have the form **Even-Even-Odd**. However, a number with an odd digit in the ones place is always odd. Since the number must be even, this case is impossible.

- Total numbers for this case = **0**.

Total Count Finally, add the counts from all possible cases.

- 125 (Case 1) + 100 (Case 2) = 225 🔢

Problem 8:

The probability that xz + y is an even number is **59/125**.

Analyze the Set and Outcomes The set of numbers is {1, 2, 3, 4, 5}.

- **Odd numbers:** {1, 3, 5} (3 choices)
- **Even numbers:** {2, 4} (2 choices) Since the three numbers x, y, z are picked with replacement, the total number of possible outcomes is:
- 5 (choices for x) × 5 (choices for y) × 5 (choices for z) = 125

Condition for an Even Result For the expression xz + y to be an even number, the two terms (xz and y) must have the same parity. This gives us two possible cases:

- **Case 1:** xz is even AND y is even.

- **Case 2:** xz is odd AND y is odd.

Calculate the Favorable Outcomes for Each Case

- **Case 1: (xz = Even) + (y = Even)**

Number of ways for y to be even: **2**

For xz to be even, at least one of x or z must be even. The easiest way to count this is (Total ways for xz) - (Ways for xz to be odd).

xz is only odd if both x and z are odd: 3 (odd choices) × 3 (odd choices) = 9 ways.

Total ways to choose x and z is 5 × 5 = 25.

So, the number of ways for xz to be even is 25 - 9 = 16.

Total outcomes for this case = 16 (for xz) × 2 (for y) = 32.

- **Case 2: (xz = Odd) + (y = Odd)**

Number of ways for y to be odd: **3**

Number of ways for xz to be odd (both x and z must be odd): 3 × 3 = 9.

Total outcomes for this case = 9 (for xz) × 3 (for y) = 27.

Calculate the Total Probability Add the outcomes from the two cases to get the total number of favorable outcomes.

32 + 27 = 59 The probability is the ratio of favorable outcomes to the total number of outcomes.

Probability = 59 / 125 🎲

Problem 9:

The value of A + B + C is **18**.

Translate to an Algebraic Equation First, we write the numbers in their algebraic form, where A, B, and C are digits.

- AB = 10A + B
- BC = 10B + C
- CA = 10C + A
- ABC = 100A + 10B + C

The equation AB + BC + CA = ABC becomes: (10A + B) + (10B + C) + (10C + A) = 100A + 10B + C

Simplify the Equation Combine the like terms on the left side:

- 11A + 11B + 11C = 100A + 10B + C

Now, move all terms to one side to find the relationship between the digits:

- 0 = (100A - 11A) + (10B - 11B) + (C - 11C)
- 0 = 89A - B - 10C
- 89A = 10C + B

Solve for the Digits The expression 10C + B is the value of the two-digit number CB. So the equation is 89A = CB.

- Since A, B, and C are digits, CB must be a two-digit number (less than 100).
- If **A = 1**, then 89 × 1 = 89. This means CB = 89, so **C = 8** and **B = 9**.
- If **A = 2**, then 89 × 2 = 178. This is a three-digit number, which is not possible for CB. The only solution is A=1, B=9, C=8.

Calculate the Final Sum

- A + B + C = 1 + 9 + 8 = 18 🧠

Problem 10:

The measure of ∠EDC is **117°**.

Analyze Triangle ABE We are given that |AB| = |BE| and the angle between these sides, ∠ABC, is **60°**. A triangle with two equal sides and a 60° angle between them is an **equilateral triangle**.

- This means all sides are equal: |AB| = |BE| = |AE|.
- It also means all angles are 60°, so ∠BAE = 60° and ∠AEB = 60°.

Analyze Triangle ADE We are given |AD| = |AB|. From Step 1, we know |AE| = |AB|. Therefore, we can conclude that |AD| = |AE|, making triangle **ADE** an **isosceles triangle**.

- We can find its vertex angle, ∠DAE: ∠DAE = ∠BAC - ∠BAE = 114° - 60° = 54°
- Now we can find the two equal base angles, ∠ADE and ∠AED: ∠ADE = ∠AED = (180° - 54°) / 2 = 126° / 2 = 63°

Find the Angles in Triangle DEC Our goal is to find ∠EDC. To do this, we can find the other two angles in triangle DEC.

- **Find ∠DCE (or ∠C):** In the main triangle ABC, the sum of angles is 180°. ∠C = 180° - ∠A - ∠B = 180° - 114° - 60° = 6°
- **Find ∠DEC:** The angle ∠AEB (which is 60°) and the angle ∠AEC lie on the straight line BC, so they sum to 180°. ∠AEC = 180° - 60° = 120° From the diagram, we can see that ∠DEC = ∠AEC - ∠AED. ∠DEC = 120° - 63° = 57°

Calculate ∠EDC Finally, using the sum of angles in triangle DEC:

∠EDC + ∠DCE + ∠DEC = 180°

∠EDC + 6° + 57° = 180°

∠EDC + 63° = 180°

∠EDC = 117° 📐

Problem 11:

The value of a is **3**.

Factor Out the Common Term The easiest way to solve this is to factor out the smallest power of 2, which is 2^{2015}, from each term on the left side of the equation.

- $2^{2018} = 2^3 \times 2^{2015}$
- $2^{2017} = 2^2 \times 2^{2015}$
- $2^{2016} = 2^1 \times 2^{2015}$
- $2^{2015} = 1 \times 2^{2015}$

Now, rewrite the equation with the common factor: $2^{2015} (2^3 - 2^2 - 2^1 + 1) = a \times 2^{2015}$

Simplify and Solve The 2^{2015} on both sides will cancel out, leaving a simple expression to solve for **a**.

$a = 2^3 - 2^2 - 2^1 + 1$

$a = 8 - 4 - 2 + 1$

$a = 3$ 💡

Problem 12:

There are **13** digits in the number.

Rewrite the Bases as Prime Powers The key to this problem is to express the bases, 125 and 64, in terms of the prime factors of 10 (which are 2 and 5).

- $125 = 5 \times 5 \times 5 = 5^3$
- $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$

Simplify the Expression Now, substitute these back into the original expression and use exponent rules.

$$125^4 \times 64^2 = (5^3)^{\wedge 4} \times (2^6)^2 = 5^{12} \times 2^{12}$$

Combine to Form a Power of 10 Since both parts of the product have the same exponent, we can combine them.

$5^{12} \times 2^{12} = (5 \times 2)^{12} = 10^{12}$

Count the Digits The number 10^{12} is a 1 followed by 12 zeros (1,000,000,000,000).

This number has a total of 12 (zeros) + 1 = 13 digits. 🔟

Problem 13:

Leah can order **100** different meal combinations.

To find the total number of combinations, we can use the multiplication principle. We just need to find the number of choices Leah has for each part of the meal and multiply them together.

Entrée Options Leah will order exactly one entrée.

- Number of choices = **5**

Appetizer Options Leah will order *at most one* appetizer. This means she can either choose one of the four available appetizers or choose to have no appetizer at all.

- Number of choices = 4 + 1 (for none) = 5

Dessert Options Similarly, she will order *at most one* dessert. She can choose one of the three available desserts or choose to have none.

- Number of choices = 3 + 1 (for none) = 4

Total Combinations Multiply the number of options for each course to get the total number of possible meal combinations.

- 5 (entrées) × 5 (appetizers) × 4 (desserts) = 100 🍽️

Problem 14:

The correct answer is **D) 4**.

Breakdown of Each Shape

The question asks how many of the five listed shapes can be formed by the overlapping region (the intersection) of two triangles. Let's analyze each one.

Equilateral Triangle: (Possible) Yes, the intersection of two triangles can easily be another triangle. A smaller equilateral triangle can be formed by overlapping two larger, identical equilateral triangles.

Square: (Possible) Yes. Imagine a square. You can form a triangle by extending the top and bottom sides until they meet at a point. You can form a second triangle by extending the left and right sides until they meet. The region where these two large triangles overlap is the original square.

Regular Hexagon: (Possible) Yes. The most famous example of this is the Star of David, which is formed by overlapping two equilateral triangles. The shape in the center of the star is a perfect regular hexagon.

Kite: (Possible) Yes. A square is a specific type of kite, so since a square is possible, a kite is also possible. A non-square kite, like a rhombus, can also be formed.

Regular Pentagon: (Not Possible) While it's possible for the intersection of two triangles to have five sides (an irregular pentagon), it is not possible to make it a **regular** pentagon. The strict requirements for all five sides to be equal and all five interior angles to be 108° cannot be met by the intersecting sides of two triangles.

Therefore, **4** of the five shapes are possible. ▲

Problem 15:

The median height of the five students is **69** inches.

Convert Averages to Sums First, let's find the total height for each group by multiplying the average by the number of students.

- **Sum of the 3 shortest students:** $3 \times 58 = 174$ inches.
- **Sum of the 3 tallest students:** $3 \times 70 = 210$ inches.
- **Sum of all 5 students:** $5 \times 63 = 315$ inches.

Find the Median Height The median height is the height of the middle student. Let's call the students' heights h_1, h_2, h_3, h_4, h_5 from shortest to tallest. The median is $\mathbf{h_3}$.

- The sum of the three shortest is $h_1 + h_2 + h_3 = 174$.
- The sum of the three tallest is $h_3 + h_4 + h_5 = 210$.

If we add these two sums together, we are summing all five students, but we are counting the middle student (h_3) twice.

- $(h_1 + h_2 + h_3) + (h_3 + h_4 + h_5) = 174 + 210 = 384$
- (Sum of all 5 students) + $h_3 = 384$

We know the sum of all 5 students is 315. Now we can find the median (h_3).

- $315 + h_3 = 384$
- $h_3 = 384 - 315 = 69$ 📏