Solution:

Problem 1:

The decimal is equal to **0.32**.

Simplify the Given Equation Let's start with the equation x + 1/y = 3.125. We can combine the terms on the left side by finding a common denominator:

- (xy/y) + (1/y) = 3.125
- (xy + 1) / y = 3.125

Find the Relationship The expression we need to find is y / (xy + 1). Notice that this expression is the **reciprocal** (the multiplicative inverse) of the expression we just derived.

Calculate the Value Since our expression is the reciprocal of 3.125, its value is 1 / 3.125. To make this calculation easier, let's convert 3.125 to a fraction.

- 3.125 = 3 1/8 = 25/8 Now, we can find the reciprocal:
- 1 / (25/8) = 8/25 Finally, convert this fraction back to a decimal:
- 8 / 25 = 32 / 100 = 0.32 💡

Problem 2:

The correct answer is **C) (10, 6)**.

Understand the Condition For a triangle to be equilateral, all three of its sides must have the exact same length. This means the three given expressions must all result in the same value.

- a + 2b
- 3a - b
- 5b - a

Test the Options We can test each pair of values for **a** and **b** given in the options to see which one fails to produce three equal side lengths.

A) (12, 8):

- 12 + 2(8) = 12 + 16 = 28
- 3(12) - 8 = 36 - 8 = 28
- 5(8) - 12 = 40 - 12 = 28 *(This works.)*

B) (9/2, 3):

- (9/2) + 2(3) = 4.5 + 6 = 10.5
- 3(9/2) - 3 = 13.5 - 3 = 10.5
- 5(3) - (9/2) = 15 - 4.5 = 10.5 *(This works.)*

C) (10, 6):

- 10 + 2(6) = 10 + 12 = 22
- 3(10) - 6 = 30 - 6 = 24
- 5(6) - 10 = 30 - 10 = 20 *(This does **not** work, as the three values are different.)*

D) (3, 2):

- 3 + 2(2) = 3 + 4 = 7
- 3(3) - 2 = 9 - 2 = 7
- 5(2) - 3 = 10 - 3 = 7 *(This works.)*

E) (3/2, 1):

- (3/2) + 2(1) = 1.5 + 2 = 3.5
- 3(3/2) - 1 = 4.5 - 1 = 3.5
- 5(1) - (3/2) = 5 - 1.5 = 3.5 *(This works.)*

The only pair of values that does not result in an equilateral triangle is **(10, 6)**.

Problem 3:

The greatest possible value of $A \times B \times C$ is **210**.

We can find the values of A, B, and C by using the divisibility rules one by one.

Divisibility by 5 The number CAB must be divisible by 5. A number is divisible by 5 if its last digit is 0 or 5. Since the digits are non-zero, the last digit, **B**, must be **5**.

Divisibility by 4 The number ABC must be divisible by 4. A number is divisible by 4 if the number formed by its last two digits is divisible by 4. We know B=5, so the number formed by the last two digits is 5C. The two-digit numbers starting with 5 that are divisible by 4 are 52 and 56. Therefore, **C** must be either **2** or **6**.

Divisibility by 9 The number BCA must be divisible by 9. A number is divisible by 9 if the sum of its digits is a multiple of 9.

The sum is B + C + A. Since we know B=5, the sum is 5 + C + A.

Combine the Clues Now we can test our two possible values for C:

Case 1: C = 2. The sum of the digits is 5 + 2 + A = 7 + A. For this to be a multiple of 9, A would have to be 2. However, the problem states the digits A, B, and C are **distinct**, so A cannot be 2 if C is 2. This case is not possible.

Case 2: C = 6. The sum of the digits is 5 + 6 + A = 11 + A. For this to be a multiple of 9, A must be **7** (since 11 + 7 = 18, and 18 is a multiple of 9). This is a valid solution, as A, B, and C are all different (A=7, B=5, C=6).

Calculate the Product We have found the unique digits: **A = 7, B = 5, C = 6**. The product is:

- $7 \times 5 \times 6 = 210$ 🧠

Problem 4:

The correct option is **E) None of preceding**. The actual number is 24.

Determine the Cube's Dimensions The large cube is made from **64** small cubes. To find its dimensions, we take the cube root of 64.

- $\sqrt[3]{64} = 4$
- This means the large cube is a **4x4x4** arrangement of smaller cubes.

Identify the 2-Faced Cubes The small cubes with paint on exactly two faces are the ones that are located along the **edges** of the large cube, but are not the corner pieces.

Calculate the Number of 2-Faced Cubes

- A cube has **12** edges.
- Each edge of the 4x4x4 cube is made of 4 small cubes. The two cubes at the ends of each edge are corner pieces (which have 3 painted faces).

- This leaves 4 - 2 = 2 cubes on each edge that have exactly two faces painted.
- Total 2-faced cubes = 12 (edges) × 2 (cubes per edge) = 24.

Since 24 is not listed in options A, B, C, or D, the correct answer is **E) None of preceding.** 🟥

Problem 5:

The empty jar weighs **4/3** pounds.

Find the Weight of the Water Poured Out The difference between the weight of the full jar and the partially empty jar is the weight of the water that was removed.

- 6 pounds - 4 pounds = 2 pounds

Calculate the Total Weight of the Water We are told that the 2 pounds of water poured out represents **3/7** of the total amount of water the jar can hold. Let **W** be the total weight of the water.

- (3/7) * W = 2
- W = 2 / (3/7) = 2 * (7/3) = 14/3 pounds. The total weight of the water when the jar is full is **14/3** pounds.

Find the Weight of the Empty Jar The weight of the empty jar is the weight of the full jar (6 pounds) minus the weight of the water inside it (14/3 pounds).

- Weight of jar = 6 - 14/3
- Weight of jar = 18/3 - 14/3 = 4/3 pounds 💧

Problem 6:

The area of the rectangle is **18** unit squares.

Analyze the Hexagon's Area A regular hexagon can be perfectly divided into **6** small, identical equilateral triangles.

- We are given that the area of one hexagon is **6** unit squares.
- This means the area of each small equilateral triangle is 6 / 6 = 1 unit square.

Visualize the Tiling The key to this problem is to see that the entire rectangle can be perfectly tiled by these same small equilateral triangles.

- The two hexagons inside the rectangle are made up of 2 × 6 = 12 of these triangles.
- The empty spaces in the four corners of the rectangle can also be filled with these triangles. A careful geometric arrangement shows that the four corner spaces are equivalent to **6** more of these equilateral triangles.

Calculate the Total Area The total area of the rectangle is the sum of the areas of the hexagons and the corner spaces, measured in our "triangle units."

- Total Triangles = 12 (from hexagons) + 6 (from corners) = 18 triangles.
- Since each triangle has an area of **1** unit square, the total area of the rectangle is **18** unit squares. 🖼️

Problem 7:

The maximum number of friends Berti has is **8.**

Set Up the Equation Let **m** be the month number (1 for January, 2 for February, etc.) and **d** be the day of the month. The condition for each birthday is:

m + d = 35 This can be rearranged to find the required day for any given month: d = 35 - m.

Test Each Month Now we can go through the months and see which ones produce a valid date. A date is only valid if the calculated day d actually exists in that month.

April (m=4): d = 35 - 4 = 31. (Invalid, April only has 30 days)
May (m=5): d = 35 - 5 = 30. (Valid, May has 31 days) -> **May 30**
June (m=6): d = 35 - 6 = 29. (Valid, June has 30 days) -> **June 29**
July (m=7): d = 35 - 7 = 28. (Valid, July has 31 days) -> **July 28**
August (m=8): d = 35 - 8 = 27. (Valid, August has 31 days) -> **August 27**
September (m=9): d = 35 - 9 = 26. (Valid, September has 30 days) -> **September 26**

October (m=10): d = 35 - 10 = 25. (Valid, October has 31 days) -> **October 25**
November (m=11): d = 35 - 11 = 24. (Valid, November has 30 days) -> **November 24**
December (m=12): d = 35 - 12 = 23. (Valid, December has 31 days) -> **December 23**

Count the Friends Counting the number of valid birthdays, we find there are **8** possible dates. Since no two friends have the same birthday, the maximum number of friends is 8. 🎂

Problem 8:

There are **18** two-digit positive integers that have at least one digit that is 8.

We can find the answer by counting all the numbers that have an 8 in the ones place and all the numbers that have an 8 in the tens place, then making sure not to double-count.

Numbers with 8 in the Ones Place These numbers are of the form _8. The first digit can be anything from 1 to 9.

- The list is: 18, 28, 38, 48, 58, 68, 78, 88, 98.
- This gives us **9** numbers.

Numbers with 8 in the Tens Place These numbers are of the form 8_. The second digit can be anything from 0 to 9.

- The list is: 80, 81, 82, 83, 84, 85, 86, 87, 88, 89.
- This gives us **10** numbers.

Find the Total If we add the two counts (9 + 10 = 19), we have counted the number **88** twice. We must subtract this overlap.

- 9 + 10 - 1 = 18

There are **18** such numbers. 🔢

Problem 9:

The number is **55**.

Translate the Problem into an Equation Let the two-digit number be represented by its digits **a** (tens place) and **b** (ones place). The value of the number is $10a + b$.

- The product of its digits is $a \times b$.
- The sum of its digits is $a + b$.

The problem states: (the number) - (product of digits) = 3 × (sum of digits). We can write this as: $(10a + b) - (a \times b) = 3 \times (a + b)$

Simplify the Equation Now, we can simplify this algebraic equation.

- $10a + b - ab = 3a + 3b$
- $7a - ab = 2b$
- $a(7 - b) = 2b$
- $a = 2b / (7 - b)$

Solve for the Digits Since **a** and **b** must be single-digit integers (from 0-9), and **a** cannot be 0, we can test possible values for **b** to find a valid solution. For **a** to be a positive integer, the denominator $(7 - b)$ must also be a positive integer, so **b** must be less than 7.

- **If b = 1:** $a = 2(1) / (7-1) = 2/6$ (Not an integer)
- **If b = 2:** $a = 2(2) / (7-2) = 4/5$ (Not an integer)
- **If b = 3:** $a = 2(3) / (7-3) = 6/4$ (Not an integer)
- **If b = 4:** $a = 2(4) / (7-4) = 8/3$ (Not an integer)
- **If b = 5:** $a = 2(5) / (7-5) = 10/2 = 5$ (This is a valid solution!)
- **If b = 6:** $a = 2(6) / (7-6) = 12/1 = 12$ (Not a single digit)

State and Verify Number The only solution that works is $a = 5$ and $b = 5$ The number is **55**.

- **Check:**
  - The number is 55. The product of digits is $5 \times 5 = 25$. The sum of digits is $5 + 5 = 10$.
  - $55 - 25 = 30$.
  - $3 \times 10 = 30$. The condition is satisfied. 🧠

Problem 10:

There are 728 whole numbers from 1 to 1000 that do not contain the digit 1.

Step-by-Step Breakdown

To find the total, we can count how many valid numbers exist for each length (1-digit, 2-digit, and 3-digit) and then add them together. The number 1000 is excluded because it contains the digit 1.

The digits we are allowed to use are {0, 2, 3, 4, 5, 6, 7, 8, 9}.

1-Digit Numbers

The positive single-digit numbers that do not contain a 1 are {2, 3, 4, 5, 6, 7, 8, 9}.

Count = 8

2-Digit Numbers

For the tens digit, we can't use 0 or 1. This leaves 8 choices {2, 3, 4, 5, 6, 7, 8, 9}.

For the ones digit, we can't use 1. This leaves 9 choices {0, 2, 3, 4, 5, 6, 7, 8, 9}.

Total 2-digit numbers = $8 \times 9 = 72$

3-Digit Numbers

For the hundreds digit, we can't use 0 or 1. This leaves 8 choices.

For the tens digit, we can't use 1. This leaves 9 choices.

For the ones digit, we can't use 1. This leaves 9 choices.

Total 3-digit numbers = $8 \times 9 \times 9 = 648$

Finally, add the counts from all the groups.

8 (1-digit) + 72 (2-digit) + 648 (3-digit) = 728 🔢

Problem 11:

The greatest of the numbers is **t**.

The easiest way to solve this is to compare the variables two at a time by looking at their expressions. All five expressions are equal to the same hidden number.

Compare t and x We know that $x + 4 = t - 3$. To make these equal, **t** must be larger than **x**. If you add 4 to x, you still need to subtract 3 from t to get the same result. This means t is 7 bigger than x ($t = x + 7$).

Compare t and y We know that $y^2 - 1 = t - 3$. We can rewrite this as $y^2 = t - 2$. This tells us that t is 2 bigger than y *squared* ($y \times y$). Since squaring a whole number makes it much larger, and t is even bigger than that square, **t** must be greater than **y**.

Compare t and z We know that $z^2 + 2 = t - 3$. We can rewrite this as $z^2 = t - 5$. This tells us that t is 5 bigger than z squared, so **t** must be greater than **z**.

Compare t and m We know that $m^2 + 12 = t - 3$. We can rewrite this as $m^2 = t - 15$. This tells us that t is 15 bigger than m squared, so **t** must be greater than **m**.

Since **t** is greater than all the other numbers, it is the greatest. 💡

Problem 12:

The correct answer is **B) Only II**.

Analyze the Condition The problem states that **a + b** is an odd number. For the sum of two integers to be odd, one of them must be **even** and the other must be **odd**.

Test Each Statement We need to check which of the three statements is true for both possibilities (a is even, b is odd; or a is odd, b is even).

Statement I: a - 2b is even The term 2b is always even (since 2 times any integer is even).

If **a is even**, the expression is even - even = even.

If **a is odd**, the expression is odd - even = odd. Since this is not always even, Statement I is false.

Statement II: a × b is even The product of two integers is even if at least one of them is even.

If **a is even**, the expression is even × odd = even.

If **b is even**, the expression is odd × even = even. Since one of the numbers is always even, the product is always even. Statement II is **true**.

Statement III: 4a + b is even The term 4a is always even (since 4 times any integer is even).

If **b is odd**, the expression is even + odd = odd.

If **b is even**, the expression is even + even = even. Since this is not always even, Statement III is false.

The only statement that is always true is **II**. 💡

Problem 13:

There are 24 ways to arrange the letters.

Treat "ANGLE" as a Single Block

The problem requires the letters "ANGLE" to appear consecutively and in that specific order. The easiest way to handle this is to treat the entire sequence "ANGLE" as a single, unbreakable block or unit.

Identify the Items to Arrange

Now, instead of arranging 8 individual letters, we are arranging a smaller set of items:

The block (ANGLE)

The letter T

The letter R

The letter I

We now have 4 distinct items to arrange.

Calculate the Number of Arrangements

The number of ways to arrange n distinct items is n! (n factorial). In our case, n = 4.

Number of ways = 4! = 4 × 3 × 2 × 1 = 24

Since the letters within the "ANGLE" block must remain in that order, we don't need to calculate any internal arrangements. 🧠

Problem 14:

The area of the triangle is **48**.

Split the Triangle into Two Right Triangles To find the area, we first need to find the height of the triangle. We can do this by drawing a line (the altitude) from the top vertex straight down to the middle of the base. In an isosceles triangle, this altitude splits the triangle into two identical right-angled triangles.

The base of each new right triangle is half of the original base: 12 / 2 = 6.

The hypotenuse of each new right triangle is one of the equal sides: **10**.

The height of the triangle (**h**) is the unknown side.

Find the Height Using the Pythagorean Theorem The Pythagorean theorem states $a^2 + b^2 = c^2$. We can use it to find the height h.

$h^2 + 6^2 = 10^2$

$h^2 + 36 = 100$

$h^2 = 100 - 36$

$h^2 = 64$

$h = 8$ The height of the triangle is **8**.

Calculate the Area Now we can use the formula for the area of a triangle: Area = (1/2) × base × height.

Area = (1/2) × 12 × 8

Area = 6 × 8 = 48

Problem 15:

There were **224** math problems on the test.

Find the Fraction of Correct Answers First, we need to find what fraction of the *total* problems Junaid answered correctly.

He attempts **3/4** of the problems. He answers **5/8** *of that amount* correctly. To find the overall fraction, we multiply these two fractions together:

(3/4) × (5/8) = 15/32 So, Junaid answered **15/32** of the total test problems correctly.

Set Up and Solve the Equation We know that this fraction (15/32) of the total number of problems (T) is equal to **105**.

(15/32) × T = 105 To find T, we can rearrange the equation:

T = 105 / (15/32)

T = 105 × (32/15)

T = 7 × 32 = 224

There were a total of **224** problems on the test.