Solution:

Problem 1:

The number that must occupy the box labeled e is **36**.

Understand the Rule The rule states that each number in the three middle boxes is the average of its left and right neighbors. This is the definition of an **arithmetic sequence**, which means there is a constant difference between each consecutive number in the five boxes.

Find the Common Difference We are given the first number (4) and the fourth number (28). Let's call the constant difference **d**.

- The first number is 4.
- The second number is 4 + d.
- The third number is 4 + 2d.
- The fourth number is 4 + 3d. We know the fourth number is 28, so we can set up an equation:
- 4 + 3d = 28
- 3d = 24
- d = 8 The common difference between each box is **8**.

Find the Value of e The box labeled **e** is the fifth number in the sequence. It comes directly after the fourth number (28). To find its value, we simply add the common difference.

- e = (fourth number) + d
- e = 28 + 8 = 36 🔢

Problem 2:

Tim will be 9 years old in **3** years.

Find Tim's Current Age

- First, we need to find Tom's age two years ago. Since he is 13 now, two years ago he was 13 - 2 = 11 years old.

- We know that two years ago, the sum of their ages was 15. So, Tim's age two years ago was 15 - 11 = 4.
- This means Tim's current age is 4 + 2 = 6 years old.

Calculate the Time Difference To find out how many years it will take for Tim to be 9, we subtract his current age from his target age.

- 9 (target age) - 6 (current age) = 3 years. 🐈

Problem 3:

There are **24** unit cubes that have paint on exactly two faces.

Determine the Cube's Dimensions The large cube is made from **64** small cubes. To find its dimensions, we take the cube root of 64.

- $\sqrt[3]{64} = 4$
- This means the large cube is a **4x4x4** arrangement of smaller cubes.

Identify the 2-Faced Cubes The small cubes with paint on exactly two faces are the ones that are located along the **edges** of the large cube, but are not the corner pieces.

Calculate the Number of 2-Faced Cubes

- A cube has **12** edges.
- Each edge of the 4x4x4 cube is made of 4 small cubes. The two cubes at the ends of each edge are corner pieces (which have 3 painted faces).
- This leaves 4 - 2 = 2 cubes on each edge that have exactly two faces painted.
- Total 2-faced cubes = 12 (edges) × 2 (cubes per edge) = 24. 🟩

Problem 4:

There are **4** different isosceles triangles with a perimeter of 17.

Set Up the Equations An isosceles triangle has two sides of equal length. Let's call the length of these two sides **a** and the length of the third side **b**. All side lengths must be integers.

- The perimeter is the sum of the sides: a + a + b = 17, which simplifies to 2a + b = 17.

- For the sides to form a valid triangle, the **Triangle Inequality Theorem** must hold: the sum of the lengths of any two sides must be greater than the third side. For an isosceles triangle, this simplifies to one crucial check: a + a > b, or **2a > b**.

Find All Possible Side Lengths We can list the integer pairs for **a** and **b** that satisfy the perimeter equation 2a + b = 17.

Equal Sides (a)	Base (b = 17 - 2a)	Side Lengths
1	15	{1, 1, 15}
2	13	{2, 2, 13}
3	11	{3, 3, 11}
4	9	{4, 4, 9}
5	7	{5, 5, 7}
6	5	{6, 6, 5}
7	3	{7, 7, 3}
8	1	{8, 8, 1}

Check for Valid Triangles Now, we apply the Triangle Inequality Theorem (2a > b) to our list of possibilities.

- {1, 1, 15}: Is 1+1 > 15? No.
- {2, 2, 13}: Is 2+2 > 13? No.
- {3, 3, 11}: Is 3+3 > 11? No.
- {4, 4, 9}: Is 4+4 > 9? No.
- {5, 5, 7}: Is 5+5 > 7? **Yes.**
- {6, 6, 5}: Is 6+6 > 5? **Yes.**
- {7, 7, 3}: Is 7+7 > 3? **Yes.**
- {8, 8, 1}: Is 8+8 > 1? **Yes.**

Count the Results There are **4** sets of side lengths that form a valid isosceles triangle with a perimeter of 17. 📐

Problem 5:

The value of A + B is **14**.

We can solve this cryptarithmetic puzzle by analyzing the addition column by column, from right to left.

The Ones Column The rightmost column shows that B + B + B results in a number that ends in 7.

- $3 \times B = ...7$ The only digit for B that works is **9**, because $3 \times 9 = 27$. This means the sum is 27, so we write down the 7 and **carry over 2** to the tens column.

The Tens Column This column shows 7 + A + A plus the carry-over from the ones column must result in a number ending in B (which we now know is 9).

- $7 + A + A + 2$ (carry) $= ...9$
- $9 + 2A = ...9$ For $9 + 2A$ to end in 9, 2A must end in 0. This gives two possibilities for A:
- $A = 0$ (since $2 \times 0 = 0$)
- $A = 5$ (since $2 \times 5 = 10$) The problem states that A and B are *different* digits. Since B is 9, both 0 and 5 are still possible values for A.

The Hundreds Column Let's test our two possibilities for A to see which one makes the hundreds column correct. The sum of this column is 7 + 5 + A plus any carry-over from the tens column, and the result must be 18.

- **Case 1: Assume A = 0.** In the tens column, $9 + 2(0) = 9$. There is no carry-over. The hundreds column sum would be $7 + 5 + 0 = 12$. This does not equal 18, so **A cannot be 0**.
- **Case 2: Assume A = 5.** In the tens column, $9 + 2(5) = 19$. We write down 9 (which matches B) and **carry over 1**. The hundreds column sum is now 7 + 5 + A + 1 (carry). $7 + 5 + 5 + 1 = 18$. This matches the result in the problem.

Find the Final Sum We have found the unique solution: **A = 5** and **B = 9**.

- $A + B = 5 + 9 = 14$ 🧠

Problem 6:

The number of positive divisors for 4N is **20**.

Let's look for a simple pattern in how the number of divisors changes.

Look at the change from N to 2N

- The number of divisors for N is **10**.
- When N is multiplied by 2, the number of divisors for 2N becomes **15**.

- The number of divisors increased by **5** (15 - 10 = 5).

Look at the change from 2N to 4N The number 4N is simply 2N multiplied by 2 again. Since we are doing the exact same thing (multiplying by 2), we can expect the pattern to continue in the same way.

- The number of divisors should increase by **5** again.

Find the Final Answer

- Number of divisors for 2N = **15**
- Increase = **5**
- Number of divisors for 4N = 15 + 5 = 20.

The number of divisors forms a simple sequence when we keep multiplying by 2:

- **N:** 10 divisors
- **2N:** 15 divisors (+5)
- **4N:** 20 divisors (+5) 🧠

Problem 7:

There are 20 squares of any size that can be formed.

To solve this, we need to count two different types of squares: those whose sides are aligned with the grid (horizontal and vertical) and those whose sides are tilted (diagonal).

1. Aligned Squares

We can count these by their size:

1×1 squares: There are 3 × 3 = 9 of these.

2×2 squares: There are 2 × 2 = 4 of these.

3×3 squares: There is 1 × 1 = 1 of these.

Total number of aligned squares = 9 + 4 + 1 = 14.

2. Tilted Squares

We can also form squares by connecting points on a diagonal.

Small tilted squares: These are the "diamond" shapes that fit inside a 2x2 grid of points. There are 4 of these.

Large tilted squares : These squares have sides that connect points with a "knight's move" (over 2, up 1). There are 2 of these on the grid.

Total number of tilted squares = 4 + 2 = 6.

3. Total Count

Finally, we add the counts of both types of squares together.

14 (aligned) + 6 (tilted) = 20 🔲

Problem 8:

Katharina has to give Julia **4** sweets.

Find the Total Number of Sweets First, find the total number of sweets they have together.

- 9 (Julia) + 17 (Katharina) = 26 sweets

Determine the Equal Amount To have the same amount, they need to split the total evenly.

- 26 sweets / 2 people = 13 sweets each

Calculate the Difference Katharina starts with 17 sweets and needs to end up with 13. The number of sweets she must give away is the difference.

- 17 - 13 = 4 sweets 🍬

Problem 9:

The 20th magic number is **440**.

Find the Pattern A magic number is the total number of dots in a rectangle with **two more rows than columns**. Let's analyze the first few magic numbers to find the general rule.

- **1st number (n=1):** A 1x3 rectangle (1 column, 1+2=3 rows). Dots = 1 × 3 = 3.
- **2nd number (n=2):** A 2x4 rectangle (2 columns, 2+2=4 rows). Dots = 2 × 4 = 8.
- **3rd number (n=3):** A 3x5 rectangle (3 columns, 3+2=5 rows). Dots = 3 × 5 = 15. The pattern for the n-th magic number is n × (n + 2).

Calculate the 20th Magic Number Using this formula, we can find the 20th magic number by setting n = 20.

- 20th Magic Number = $20 \times (20 + 2) = 20 \times 22 = 440$ 🔢

Problem 10:

The area of this figure is **60 cm^2**.

We can find the area of this L-shaped figure by splitting it into two smaller, simpler rectangles.

Split the Shape Imagine a horizontal line extending from the corner where the 4 cm and 10 cm sides meet. This divides the shape into a top rectangle and a bottom rectangle.

Calculate the Area of the Top Rectangle

- The width of the top rectangle is given as **5 cm**.
- The height of the top rectangle is given by the vertical side of **4 cm**.
- Area = 5 cm × 4 cm = 20 cm^2.

Calculate the Area of the Bottom Rectangle

- The width of the bottom rectangle is given as **10 cm**.
- Its height is the total height of the figure (8 cm) minus the height of the top rectangle (4 cm). Height = 8 cm - 4 cm = 4 cm.
- Area = 10 cm × 4 cm = 40 cm^2.

Find the Total Area Add the areas of the two smaller rectangles together.

- 20 cm^2 + 40 cm^2 = 60 cm^2 🖼️

Problem 11:

Denis has the longest stride.

Explanation

To cover the same distance, the person with the longest stride will need to take the **fewest** number of steps.

Let's look at how many strides each person took:

- **Denis:** 12 strides
- **Ivo:** 14 strides
- **Anni:** 15 strides
- **Betty:** 17 strides

Since **Denis** took the smallest number of strides, he must have the longest stride. 👟

Problem 12:

The number that must replace the question mark is **99**.

Analyze the Relationship Between the Rows The problem states that the total of the numbers in each row is the same. Let's compare the first 10 numbers in each row.

- **Top Row:** 1, 2, 3, ..., 10
- **Bottom Row:** 11, 12, 13, ..., 20 Notice that each number in the bottom row is exactly **10 greater** than the number directly above it.

Calculate the Difference in Sums Since there are 10 such pairs, the sum of the first 10 numbers in the bottom row is $10 \times 10 = 100$ greater than the sum of the first 10 numbers in the top row.

Find the Missing Number To make the total sum of both rows equal, the last number in the bottom row must be **100 less** than the last number in the top row to balance out this difference.

- Last number in the top row = 199
- Missing number = 199 - 100 = 99 🔢