Grade 9

Problem №1.

Let x, y, and z be real numbers satisfying the system of equations:

$$xy + 4z = 60$$
$$yz + 4x = 60$$
$$zx + 4y = 60.$$

Let S be the set of possible values of x .Find the sum of the squares of the elements of S.

Problem №2.

A lake contains 250 trout, along with a variety of other fish. When a marine biologist catches and releases a sample of 180 fish from the lake, 30 are identified as trout. Assume that the ratio of trout to the total number of fish is the same in both the sample and the lake. How many fish are there in the lake?

Problem №3.

The figure below shows a large white circle with a number of smaller white and shaded circles in its interior. What fraction of the interior of the large white circle is shaded?
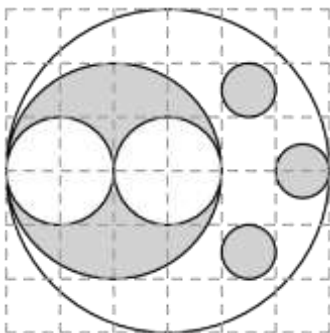
Problem №4.

If we list all 3-digit numbers on a piece of paper, how many numbers do we write down that are divisible by 11?

Problem №5.

In a geometric sequences of positive terms, the difference between the 6th and the 5th terms is 324, and the difference between the 2nd and 1st terms is 4. What is the sum of the first 6 terms of the sequence?

Problem №6.

Alina writes the numbers 1, 2, …, 9 on separate cards, one number per card. She wishes to divide the cards into 3 groups of 3 cards so that the sum of the number in each group will be the same. In how many ways can this be done?

Problem №7.

In a sequence of positive integers, each term after the second is the product of the previous two terms. The sixth term in the sequence is 4000. What is the first term?

Problem №8.

Isosceles $\triangle ABC$ has equal side lengths AB and BC. In the figure below, segments are drawn parallel to $\overline{AC}$ so that the shaded portions of $\triangle ABC$ have the same area. The heights of the two unshaded portions are 11 and 5 units, respectively. What is the height of h of $\triangle ABC$?