Grade 11

Problem №1.

Let P be a point on the circumcircle of square $ABCD$ such that $PA \cdot PC = 56$ and $PB \cdot PD = 90$. What is the area of square $ABCD$?

Problem №2.

From the basement of a house to its ground floor there are 7 stairs. Hoppy the Frog can hop either 1 or 2 stairs at a time, and he never hops backwards.

In how many ways can Hoppy the Frog reach the ground floor from the basement?

Problem №3.

Ted mistakenly wrote $2^m \cdot \sqrt{\frac{1}{4096}}$ as $2 \cdot \sqrt[m]{\frac{1}{4096}}$. What is the sum of all real numbers m for which these two expressions have the same value?

Problem №4.

The sum $\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \cdots + \frac{2021}{2022!}$ can be expressed as $a - \frac{1}{b!}$, where a and b are positive integers. What is $a+b$?

Problem №5.

Rhombus $ABCD$ has $\angle BAD < 90^\circ$. There is a point P on the incircle of the rhombus such that the distances from P to lines DA, AB and BC are 9, 5, and 16, respectively. Find the perimeter of $ABCD$.

Problem №6.

Find the number of positive integers less than or equal to 2017 that are divisible by 5 but are not divisible by either 7 or 11.

Problem №7.

When three positive integers are added in pairs, the resulting sums are 998, 1050 and 1234. What is the difference between the largest and smallest of the three original positive integers?

Problem №8.

The five-digit positive integer is written in the form $\overline{12ABC}$ where A, B, and C are unknown digits.

If this five-digit integer is a perfect square that is divisible by both 9 and 4, what is the value of $A+B+C$?