Grade 10

Problem №1.

If $\sqrt{\log_b n} = \log_b \sqrt{n}$ and $b\, log_b\, n = log_b\, bn$ then the value of n is equal to $\frac{j}{k}$, where j and k are relatively prime. What is $j+k$?

Problem №2.

Call a positive integer n **extra-distinct** if the remainders when n is divided by 2, 3, 4, 5 and 6 are distinct. Find the number of extra-distinct positive integers less than 1000.

Problem №3.

Five men and nine women stand equally spaced around a circle in random order. The probability that every man stands diametrically opposite a woman is $\frac{m}{n}$,where m and n are relatively prime positive integers. Find $m+n$.

Problem №4.

Find the number of cubic polynomials $p(x) = x^3 + ax^2 + ax + c$, where

a, b, and c are integers in $\{-20, -19, -18, \ldots, 18, 19, 20\}$, such that there is a unique integer $m \neq 2$ with $p(m) = p(2)$.

Problem №5.

Fifteen integers $a_1, a_2, a_3, \ldots, a_{15}$ are arranged in order on a number line. The integers are equally spaced and have the property that $1 \leq a_1 \leq 10$, $13 \leq a_2 \leq 20$, $241 \leq a_{15} \leq 250$. What is the sum of digits of a_{14}?

Problem №6.

The number of apples growing on each of six apple trees form an arithmetic sequence where the greatest number of apples growing on any of the six trees is double the least number of apples growing on any of the six trees. The total number of apples growing on all six trees is 990. Find the greatest number of apples growing on any of the six trees.

Problem №7.

A **perfect cube** is a number that is obtained by multiplying the same integer by itself three times. For example, multiplying the number 5 by itself three times results in $5 \times 5 \times 5 = 5^3$

What is the **smallest positive integer k** for which the integer

$2^5 \times 3^4 \times 5^2 \times 7^2 \times k$ is a perfect cube?

Problem №8.

The sum of positive odd integers from **1** to **k** both included, is 15376. Find the value of **k**.