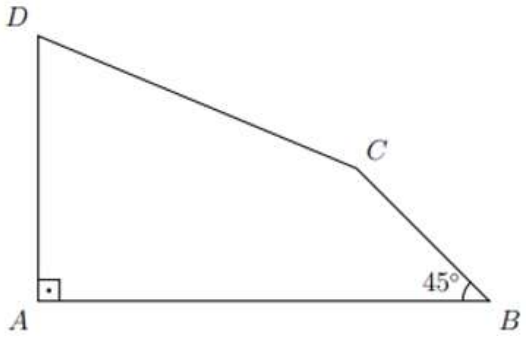
Grade 9

Problem №1.

$ABCD$ is a quadrilateral with $AB \perp AD. m\angle ABC = 45°$, $BC = 5\sqrt{2}$, $AD = 10$ and $AB = 17$. Find DC.



A) $8\sqrt{2}$ B) 12 C) $9\sqrt{2}$ D) 13 E) None of the preceding

Problem №2.

How many digits are in the number $125^4 \cdot 64^2$?

A) 10 B) 11 C) 12 D) 13 E) 14

Problem №3.

What positive integer n satisfies the following equation?

$$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\ldots\left(1-\frac{1}{n^2}\right)=\frac{n+1}{16}$$

A) 4 B) 8 C) 9 D) 12 E) 16

Problem №4.

Let $N = 12345678910111213 \ldots 394041$ be the 73-digit number obtained by writing the integers from 1 to 41 in order, one after the other. What is the remainder when N is divided by 9?

A) 1 B) 5 C) 6 D) 7 E) 9

Problem №5.

The sides of a triangle have lengths 7, 14, and c. For how many positive *n* integer values of *c* is the triangle acute?

A) 2 B) 3 C) 4 D) 5 E) 8

Problem №6.

The sum of three positive integers is 532. How many zeros, at most, does the product of those three numbers end with?

A) 7 B) 6 C) 5 D) 4 E) 3

Problem №7.

How many positive integers less than 2019 are multiples of 3 or 4 but not 5?

A) 702 B) 807 C) 917 D) 927 E) None of the preceding

Problem №8.

How many real number solutions does the equation $5^x x^2 + 125 = 5^{x+2} + 5x^2$ have?

A) 0 B) 1 C) 3 D) 5 E) Infinitely many

Problem №9.

What is the minimum value of $x + y + z + t$ where *x, y, z* and *t* are positive integers satisfying $3^{8x} + 3^{5y} + 3^{12z} = 3^{19t}$

A) 162 B) 166 C) 170 D) 174 E) 180

Problem №10.

In how many ways can a blank 3×3 grid be filled with the integers from 1 to 9 so that squares containing consecutive integers are adjacent (i.e., have a common edge)? Hint: Of the three examples below, A and B satisfy the given conditions, while C does not because the squares containing 1 and 2 are not adjacent!

9	8	7
2	1	6
3	4	5

A

7	8	9
6	5	4
1	2	3

B

1	3	2
6	5	4
7	8	9

C

A) 20 B) 32 C) 40 D) 50 E) 60

Problem №11.

Let **A, B,** and **C** be three distinct, non-zero digits. Find their values in the following equation:

AB + BC + CA = ABC

What is the value of **A + B + C**?

A) 12 B) 14 C) 15 D) 18 E) 19

Problem №12.

How many 4-digit positive integers containing only odd digits are divisible by 5?

A) 100 B) 125 C) 250 D) 500 E) 625

Problem №13.

Rhombus $ABCD$ has $AB = 6$ and $ABC = 60°$. Point P is chosen in order to minimize the sum of the distances from P to each of the points A, B, C and D. What is $PA + PB + PC + PD$?

A) $3+3\sqrt{3}$ B) 6 C) $6+3\sqrt{3}$ D) $6+6\sqrt{3}$ E) 18

Problem №14.

Richard's average bowling score in a league is 180. Today, he bowled three league games and scored 193, 207 and 200, which raised his average to 182. Including the games bowled today, how many league games has Richard bowled thus far?

A) 20 B) 27 C) 30 D) 33 E) 36

Problem №15.

Alice writes down a list of whole numbers beginning with 1. To generate the next number in the list, she either adds 6 to the previous number, or she multiplies the previous number by 4. For example, her list could be the sequence 1, 7, 28, 34, 40, 160, … . Which of the following numbers cannot appear in Alice's list?

A) 109 B) 151 C) 244 D) 335 E) 412