Grade 11

Problem №1.

If $\log_2 3 = a$ and $\log_3 5 = b$, what is the value of $\log_2 60$ in terms of a and b?

A) $2b + a$ B) $a + b - 1$ C) $ab + a + b + 2$ D) $ab + 2$ E) None of the preceding

Problem №2.

How many integer values for n make $n^{18/n}$ have an integer value?

A) 6   B) 7   C) 8   D) 9   E) 12

Problem №3.

The sum of the first 20 terms of an arithmetic sequence which has first term 1 is equal to the sum of the first 10 terms of an arithmetic sequence which has first term 10. If positive integers x and y are the respective common differences of the sequences, what is the minimum value of $x + y$?

A) 35   B) 38   C) 40   D) 43   E) None of the preceding

Problem №4.

What is the remainder when $1^4 + 2^4 + 3^4 + \ldots + 2019^4$ is divided by 16?

A) 2   B) 4   C) 6   D) 8   E) 14

Problem №5.

Let x, y, and z be three numbers randomly picked with replacement from the set $\{1, 2, 3, 4, 5\}$, what is the probability that $xz + y$ is even number?

A) $\frac{2}{5}$ B) $\frac{23}{25}$ C) $\frac{39}{125}$ D) $\frac{64}{125}$ E) $\frac{59}{125}$

Problem №6.

Suppose x and y are real numbers that satisfy $2x^2 - 3y = -\frac{17}{2}$ and $y^2 - 4x = 7$. Find the value of $x + y$.

A) $\frac{7}{2}$ B) $\frac{5}{4}$ C) $\frac{3}{2}$ D) $\frac{1}{4}$ E) None of the preceding

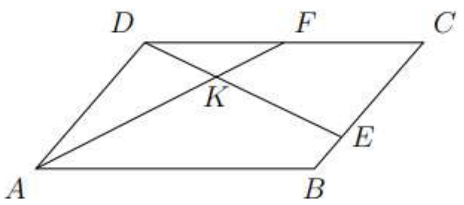
Problem №7.

Suppose you have 98 identical real coins and 3 identical fake coins that look like real coins but are lighter in weight. By using only an equal-arm scale, what is the least number of weighings required to find 25 real coins?

A) 2 B) 3 C) 4 D) 5 E) None of the preceding

Problem №8.

ABCD is a parallelogram. E and F are two points on BC and CD, respectively. If CE=3BE, $CF = DF$, $DE \cap AF = \{K\}$ and $KF = 6$, find AK.



A) 10 B) 12 C) 14 D) 15 E) 16

Problem №9.

Suppose a and b are real numbers such that $ab^2 = 1$ and $a^3 + 3b^3 = 4$. What is the product of all possible values of $a^3 + b^3$.

A) 24 B) 25 C) 27 D) 39 E) 44

Problem №10.

An *EZ* number is defined as any positive integer with the following properties:

- ◆ It has at least two digits.
- ◆ All its digits are the same.

◆ It has exactly 4 positive divisors

For example: $111 = 3 \times 37$ is an *EZ* number. How many *EZ* number less than 10^5?

A) 5 B) 6 C) 7 D) 8 E) 9

Problem №11.

Suppose a and b represent positive numbers. Of the two numbers, a is the smaller and b the larger. What number represents the point two third of the way between a and b on a number line?

A) $\frac{a+b}{3}$ B) $\frac{a+2b}{3}$ C) $\frac{3a+b}{3}$ D) $\frac{2a+3b}{3}$ E) None of the preceding

Problem №12.

In a convex pentagon $ABCDE$, $m\angle A = 40°$, $m\angle B = m\angle E$ and $m\angle C = m\angle D$. What is the sum of the measures of $\angle B$ and $\angle C$?

A) 225° B) 230° C) 240° D) 250° E) cannot be determined

Problem №13.

If $x_1 < x_2 < \ldots < x_n$ are whole numbers, for some positive integer n, such that

$$2^{x_1} + 2^{x_2} + \ldots + 2^{x_n} = 160000.$$

Find the value of $x_1 + x_n$.

A) 36 B) 32 C) 28 D) 25 E) 24

Problem №14.

Suppose BD bisects $\angle ABC$, $BD = 3\sqrt{5}$, $AB = 8$, and $DC = \frac{3}{2}$.

Find $AD + BC$

A) 8 B) 7 C) 6 D) 5 E) 4

Problem №15.

As n ranges over all positive integers, how many distinct values can be found for the greatest common divisor of $6n + 15$ and $10n + 21$?

A) 2 B) 3 C) 4 D) 5 E) 6