Grade 10

Problem №1.

If the function f satisfies $f\left(x-\frac{1}{x}\right)=x^2+\frac{1}{x^2}$ for every positive real x, what is the value of $f(2)$?

A) $\frac{1}{2}$ B) $\frac{3}{2}$ C) 3 D) 4 E) 6

Problem №2.

Start with 243. In each blank below, insert either ×3 or ÷9 to create a true equation. How many different, true equations can be formed?

243 ____ ____ ____ ____ ____ ____ = 1

A) 5 B) 12 C) 35 D) 128 E) None of the preceding

Problem №3.

A parabola with equation $y = ax^2 + bx + c$ has vertex (h, k). How many of the six quantities *a, b, c, h, k* and $\Delta = b^2 - 4ac$ can be negative at the same time?

A) Most 2 B) At most 3 C) At most 4 D) At most 5 E) All six

Problem №4.

An integer N has 10 positive divisors. If $2N$ has 15 positive divisors and $3N$ has 20 positive divisors, how many positive divisors does $4N$ have?

A) 15 B) 20 C) 34 D) 28 E) 32

Problem №5.

Find the number of integer solutions to the following equation $(x^2-3x+1)^{x+1}=1$

A) 0 B) 1 C) 2 D) 3 E) 4

Problem №6.

What is the number of ordered pairs (x, y) of positive integers that satisfy the equation $2x + 3y = 120$?

A) 19 B) 24 C) 29 D) 36 E) None of the preceding

Problem №7.

In triangle ABC, the medians BE and CD intersect at F and are perpendicular to each other. If $BE = 18$ and $CD = 24$, find the length AF .

A) 15 B) 20 C) 25 D) 30 E) None of the preceding

Problem №8.

What is the maximum possible value of $x + y$ for positive integers x and y that satisfies the following equation?

$$log_2(log_{2^x}(log_{2^y}2^{400})) = 0$$

A) 53 B) 102 C) 201 D) 400 E) None of the preceding

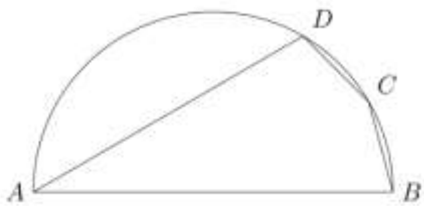
Problem №9.

Both FIZ and $FIZZ$ are acceptable $Scrabble$ words, though the version with two Z's is better known. How many six-letter strings formed only from the letters F, I, and Z will contain the word FIZ but not the word $FIZZ$?

A) 64 B) 72 C) 80 D) 96 E) None of the preceding

Problem №10.

On a semicircle with diameter AB, two points C and D are taken such that BC=CD=2. If AB=6, find the value of 6AD.



A) 27 B) 28 C) 29 D) 30 E) 31

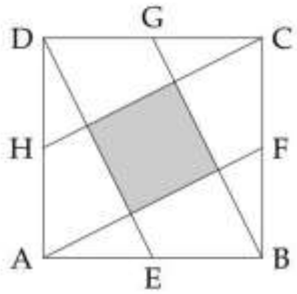
Problem №11.

Robert has a coin which lands heads with probability $\frac{2}{3}$ and tail with probability $\frac{1}{3}$. He flips this coin three times. What is the probability that he obtains at least one tail?

A) $\frac{4}{9}$ B) $\frac{17}{27}$ C) $\frac{19}{27}$ D) $\frac{23}{27}$ E) $\frac{26}{27}$

Problem №12.

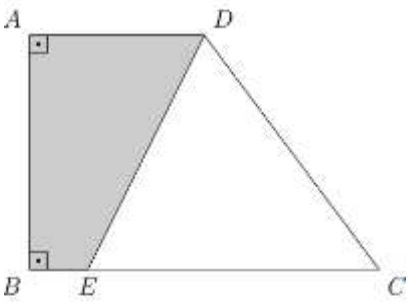
Square $ABCD$ has side length 4. Points E, F, G and H are the midpoints of AB, BC, CD and DA respectively. What is the area of the shaded square?



A) 2.4 B) 3.2 C) 3.6 D) 4 E) None of the preceding

Problem №13.

$ABCD$ is a trapezoid with $AD // BC$, $AB \perp AD$, $AB = 8$, $AD = 6$ and $DC = EC = 10$. Find the area of $ABED$?



A) 24 B) 26 C) 28 D) 30 E) 32

Problem №14.

Find the value of x for which $100^{x} \times 1000^{2x} = 10000^{10}$

A) 3 B) 4 C) 5 D) 6 E) 7

Problem №15.

The cubic polynomial $x^3 + 9x^2 + 24x + 16$ has exactly two distinct real roots, both of which are integers. What is the sum of these two roots?

A) -9 B) -5 C) 2 D) 5 E) 17