Solution:

Problem 1:

As there is the same number left over (1) when removing eggs 2, 3, 4, 5, or 6 at a time, we need to first find the common multiples of 2, 3, 4, 5, and 6.

The lowest common multiple of these numbers is 60:

$$LCM(2,3,4,5,6)=60$$

Therefore, there may be 1 more egg in the cart than one of the common multiples. Thus, the number of eggs in the cart may be 61, 121, 181, 241, 301, 361, 421, or 481 (as the number of eggs in the cart cannot be more than 500).

As the number must be a multiple of 7, the only possible answer is 301 as $301=43\times7$. Therefore, Mike the merchant has 301 eggs in his cart.

ANS: 301

Problem 2:

Writing the final price of 31 dollars and 93 cents in cents only gives 3193 cents, which is the product of the price of a single pack of trail mix in cents and the number of packs purchased.

As the prime factorization of 3193 is 3193=31×1033193=31×103 and the price of a single pack is less than 50 cents, we know that the price must have been 31 cents and the number of packs purchased must have been 103.

ANS: 103

Problem 3:

The bus takes 7 trips on that day, which means the total number of tourists were 100+96+92+88+84+80+76=616.

ANS: 616

Problem 4:

They would represent our 100 as **144**.

The Xenon inhabitants use a number system with eight digits (0-7), which is known as a **base-8** or **octal** system. Our standard system uses ten digits (0-9) and is called **base-10**. To solve the problem, we just need to convert the base-10 number 100 into its base-8 equivalent.

The simplest way to convert from base-10 to base-8 is by using repeated division. We divide the number by 8 and record the remainders until the result is zero.

1. Divide 100 by 8: 100÷8=12 with a remainder of **4**.
2. Now, divide the result (12) by 8: 12÷8=1 with a remainder of **4**.
3. Finally, divide that result (1) by 8: 1÷8=0 with a remainder of **1**.

To get the final answer, you read the remainders from the bottom up. This gives you **144**.

Problem 5:

The smallest five-digit number that meets the conditions is **12,384**.

The number must satisfy three conditions: it's a five-digit number, its digits are all different, and it's divisible by each of these digits. Here are a few key deductions from these rules:

- The digit **0 cannot be used**, as division by zero is impossible.
- If the digit **5** were used, the number would have to end in 5. However, if any even digit (2, 4, 6, 8) were also used, the number would have to be even. A number can't be both. To find the smallest number, it's best to use the smaller digits, so we'll exclude 5.
- If the digit **3** is used, the sum of the digits must be divisible by 3.

1. **Find the Right Set of Digits** To make the number as small as possible, we should use the smallest available digits. Since we're excluding 0 and 5, let's try to build a set. To satisfy the divisibility rule for 3, the sum of the five digits must be a multiple of 3. The set of digits with the smallest possible sum that is a multiple of 3 is **{1, 2, 3, 4, 8}**, as their sum is 18.
2. **Apply Divisibility Rules** The number must be divisible by 1, 2, 3, 4, and 8.

Divisibility by 3: Guaranteed, as the sum is 18.

Divisibility by 8: This is the strictest rule here, as it also guarantees divisibility by 2 and 4. The last three digits of the number must form a number that is divisible by 8.

3. **Find the Smallest Arrangement** We need to arrange the digits {1, 2, 3, 4, 8} into the smallest possible number that follows the rules.
   The number must start with **1** to be as small as possible.
   Let's try creating the smallest number (12...) while ensuring the last three digits are divisible by 8.
   The number **12,384** uses all the required digits. Let's check it:
      Divisible by 1, 2, 3? Yes.
      Divisible by 4? Yes, because the last two digits, 84, are divisible by 4.
      Divisible by 8? Yes, because the last three digits, 384, are divisible by 8 ($384 \div 8 = 48$).

Since this is the first valid number we find when building from the smallest digits upwards, it is the correct answer.

Problem 6:

The largest possible perimeter of Natalie's original unfolded piece of paper is **48 cm**.

To solve this, we need to work backward by "unfolding" the paper. Each time we unfold the paper, one of its dimensions doubles. Since the paper was folded twice, we need to account for the different ways the folds could have been made. The largest perimeter will come from the original shape that is the longest and skinniest.

The twice-folded paper is a 4 cm by 5 cm rectangle. There are two main ways the original paper could have been folded:

1. **Folding in the Same Direction Twice:** Imagine folding a long strip of paper in half, and then in half again the same way.

Possibility A: The 4 cm side was created by folding twice. Unfolding it twice would make its length $4 \times 2 \times 2 = 16$ cm. The original paper would be **16 cm by 5 cm**.

Possibility B: The 5 cm side was created by folding twice. Unfolding it twice would make its length $5 \times 2 \times 2 = 20$ cm. The original paper would be **4 cm by 20 cm**.

2. **Folding in Different Directions:** Imagine folding the paper in half one way, then rotating it 90 degrees and folding it in half again.

Possibility C: Both the 4 cm and 5 cm sides were created by a single fold. Unfolding both sides once would make the dimensions (4×2) by (5×2). The original paper would be **8 cm by 10 cm**.

Finding the Largest Perimeter

Now, we calculate the perimeter $(2 \times (\text{length} + \text{width}))$ for each possible original shape:

- Perimeter of 16 cm by 5 cm paper = $2 \times (16 + 5) = 42$ cm.
- Perimeter of 8 cm by 10 cm paper = $2 \times (8 + 10) = 36$ cm.
- Perimeter of 4 cm by 20 cm paper = $2 \times (4 + 20) = 48$ cm.

Comparing the results, the largest possible perimeter is **48 cm**.

Problem 7:

The value of **x** is **11**.

The goal is to simplify the left side of the equation until it matches the format of the right side, which is number × 45^x. Once both sides are in a similar format, we can easily solve for **x**.

Factor the Numerator The first step is to simplify the top part of the fraction by factoring out the lowest power of 45, which is 45^{11}.

$$45^{13} - 45^{11} = 45^{11}(45^2 - 1)$$

Simplify the Expression Now, calculate the value inside the parentheses.

$$45^2 - 1 = 2025 - 1 = 2024$$

This makes the left side of the equation:

$$\frac{45^{11} \cdot 2024}{22}$$

1. **Perform the Division** Next, divide the numbers.

2024÷22=92

The entire left side of the equation simplifies to: 92·4511

2. **Solve for x** Finally, we set the simplified left side equal to the original right side.

$$92 \cdot 45^{11} = 92 \cdot 45^x$$

By comparing both sides of the equation, it's clear that **x = 11**.

Problem 8:

A total of **45** different chords can be formed.

A chord is a line segment created by connecting any two points on a circle. Since the order in which you pick the two points doesn't matter (a chord from point 1 to point 5 is the same as a chord from point 5 to point 1), this is a classic **combinations** problem. We need to find out how many ways we can choose 2 points from a total of 10.

There are two straightforward ways to solve this:

1. The Combinations Formula

We are choosing groups of **2** points from a set of **10**. The formula for this is "10 choose 2", or $\binom{10}{2}$.

$$\binom{10}{2} = \frac{10 \times 9}{2 \times 1} = \frac{90}{2} = 45$$

2. Logical Summation

You can also think about it by starting at one point and drawing lines.

- The **1st** point can connect to the 9 other points, creating **9** chords.

- The **2nd** point can connect to the 8 *remaining* points (since its chord to the 1st point is already counted), creating **8** new chords.
- The **3rd** point can connect to the 7 remaining points, creating **7** new chords.
- ...and so on, until the 9th point connects to the last one.

The total is the sum of this pattern: $9+8+7+6+5+4+3+2+1=45$