



Second Round 2021-2022

### Solution:

#### Problem 1:

When the larger wheel makes 10 full revolutions, the smaller wheel will have made **30 revolutions**.

The key to solving this is understanding that both wheels travel the same distance along the ground. Since the smaller wheel has a smaller circumference, it must spin more times to cover the same distance as the larger wheel. The number of revolutions is directly related to the ratio of their diameters.

- **Large Wheel Diameter:** 54 inches
- **Small Wheel Diameter:** 18 inches
- 1. **Find the Ratio:** First, find out how many times larger the big wheel is compared to the small wheel. We do this by dividing their diameters.

$$\frac{\text{Large wheel diameter}}{\text{Small wheel diameter}} = \frac{54 \text{ inches}}{18 \text{ inches}} = 3$$

This tells us the large wheel is **3 times larger** than the small wheel.

- 2. **Apply the Ratio to Revolutions:** Because the small wheel is 3 times smaller, it must rotate **3 times** for every single rotation of the large wheel to keep up.
- 3. **Calculate the Final Answer:** Since the large wheel makes 10 revolutions, we multiply that by our ratio of 3.

$$10 \text{ revolutions} \times 3 = 30 \text{ revolutions}$$

So, while the large wheel turns 10 times, the small wheel turns 30 times to cover the exact same distance.

#### Problem 2:

Sam has a total of **36 coins** in his piggy bank.

The key to this problem is that there's an **equal number** of nickels, dimes, and quarters. This lets us group the coins into identical "sets," with each

set containing one of each coin type. By finding the value of one set, we can figure out how many sets are needed to reach the total value of \$4.80.

- **Value of one set:** \$0.05 (nickel) + \$0.10 (dime) + \$0.25 (quarter) = \$0.40
- **Total value:** \$4.80
- 1. **Find the Value of One Set:** First, calculate the combined value of one nickel, one dime, and one quarter.  $0.05+0.10+0.25=\$0.40$
- 2. **Find the Number of Sets:** Divide the total amount of money Sam has by the value of a single set to see how many sets there are.

$$\frac{\$4.80}{\$0.40} = 12$$

- 3. So, Sam has **12 sets** of coins.

Since each set contains 3 coins, multiply the number of sets by 3 to find the total number of coins.  $12 \text{ sets} \times 3 \text{ coins per set} = 36 \text{ coins}$

### Problem 3:

The area of the lot is **8,160 square meters**.

To find the area of a rectangle, you multiply its two sides (Area = Length  $\times$  Width). First, we need to calculate the length of the second side based on the word problem.

Interestingly, the side described as "longer" is mathematically shorter than the given 120-meter side. However, this doesn't change how we calculate the area; we simply multiply the two side lengths we find.

- **Side 1:** 120 m
- **Side 2:** 68 m
- 1. **Find the Length of the Second Side:** We'll break down the description step-by-step.
  - The shorter side is **120 m**.
  - One-third of the shorter side is:  $120/3=40$  m.
  - Twice that amount is:  $2 \times 40=80$  m.
  - 12 meters less than that is:  $80-12=68$  m.
- 2. **Calculate the Area:** Now, multiply the lengths of the two sides to find the area.  $120 \text{ m} \times 68 \text{ m} = 8,160 \text{ m}^2$

#### Problem 4:

The number that goes into the leftmost box is **888**.

The key to solving this puzzle is the rule that the sum of any three consecutive boxes is **2005**. This rule creates a simple repeating pattern: the number in any box is the same as the number three boxes away from it.

- **Rule:**  $\text{Box } 1 + \text{Box } 2 + \text{Box } 3 = 2005$
- **Also:**  $\text{Box } 2 + \text{Box } 3 + \text{Box } 4 = 2005$
- **Conclusion:** This means **Box 1 = Box 4**.

This pattern ( $\text{Box } N = \text{Box } N+3$ ) repeats throughout the entire row.

**Find the Pattern:** Let's label the boxes from left to right as Box 1, Box 2, Box 3, and so on. The rule creates a repeating sequence where every third number is the same.

**Box 1 = Box 4 = Box 7**

1. **Use the Given Numbers:** The image shows that the number in **Box 7** is **888**.
2. **Solve for the First Box:** Since Box 1 is equal to Box 7, the number in the leftmost box must also be **888**.

#### Problem 5:

Mr. Friedman bought **103 packs** of trail mix.

To solve this, we need to find two numbers that multiply together to equal the total cost in cents. One number is the quantity of packs, and the other is the new sale price. The key is that the new sale price must be less than the original 50 cents.

- **Total Cost:** \$31.93, which is **3193 cents**.
  - **Equation:**  $\text{Number of Packs} \times \text{Sale Price} = 3193$
  - **Constraint:** The sale price must be less than 50 cents.
1. **Find the Factors:** We need to find the factors of the total cost, 3193. The prime factors of 3193 are **31** and **103**. This gives us two possible pairs for the sale price and quantity:
    - 31 cents and 103 packs
    - 103 cents and 31 packs
  2. **Apply the Constraint:** The sale price must be **less than 50 cents**.
    - The price of 103 cents is too high.

- The price of **31 cents** fits the condition perfectly.
- 3. **Determine the Quantity:** If the sale price was 31 cents per pack, then the number of packs he bought is found by dividing the total cost by the price.

3193 cents:31 cents/pack =**103 packs**.

#### **Problem 6:**

The shuttle bus took a total of **616 tourists** that day.

To find the total, we first need to figure out how many bus trips were made. Then, we can calculate the number of tourists on each individual trip and add them all together. The number of tourists on each trip forms a simple arithmetic sequence.

1. **Determine the Number of Trips:** The bus leaves hourly from 10:00 to 16:00, inclusive. The departure times are: 10:00, 11:00, 12:00, 13:00, 14:00, 15:00, and 16:00. This makes a total of **7 trips**.
2. **List the Tourists per Trip:** The first trip had 100 tourists, with each following trip having 4 fewer people.
  - **10:00 Trip:** 100 tourists
  - **11:00 Trip:** 96 tourists
  - **12:00 Trip:** 92 tourists
  - **13:00 Trip:** 88 tourists
  - **14:00 Trip:** 84 tourists
  - **15:00 Trip:** 80 tourists
  - **16:00 Trip:** 76 tourists
3. **Sum the Total Number of Tourists:** Adding the number of tourists from all 7 trips gives us the total for the day.

$$100+96+92+88+84+80+76=616$$

#### **Problem 7:**

Each tire was used for **24,000 kilometers**.

The key to this problem is realizing that while the car travels 30,000 km, the total "wear and tear" is distributed across all five tires. Since only four tires are used at any given moment, each of the five tires is on the car for exactly four-fifths ( $\frac{4}{5}$ ) of the total journey.

You can solve this in two simple ways:

### Total Kilometers Used

1. First, find the total distance covered by all the tires on the road combined. Since 4 tires are always running, this is:  
 $30,000 \text{ km} \times 4 \text{ tires} = 120,000 \text{ tire-km}$
2. Next, divide this total usage evenly among the 5 available tires:

$$\frac{120000 \text{ tire} - \text{km}}{5 \text{ tires}} = 24000 \text{ km per tire}$$

### Problem 8:

The value of  $K + N$  is 3.

In Row 2 (T, 2, J, K), J must be 4 (because Column 3 needs a 4, and Row 2 already has a 2).

With J=4, Row 2 (T, 2, 4, K) needs the numbers 1 and 3. Since Column 1 already has a 1, T must be 3.

This leaves only one possibility for K in that row. Therefore,  $K = 1$ .

Now, look at Column 4 (H, K, N, R). We know R must be 4 (to complete the main diagonal) and we just found  $K=1$ . The column now contains H, 1, N, 4.

Column 4 still needs the numbers 2 and 3. Since Row 3 (L, M, 3, N) already contains a 3, N cannot be 3. Therefore,  $N = 2$ .

Adding them together:  $K+N=1+2=3$ .