ARCHIMEDES MATH COMPETITION

Second Round 2021-2022

Solution:

Problem 1:

Changing all measurements into centimeters, we have the following information:

- Height of ceiling: 265 centimeters
- Light bulb's drop from the ceiling: 12 centimeters
- Emily's reach above the top of her head: 47 centimeters
- Emily's height: 143 centimeters
- Height of the stool required: NN centimeters

With all the measurements, the height of the stool must be $N=265-12+47+143=63N=265-12+47+143=63$ centimeters.

ANS: 63 cm

Problem 2:

The sum of their ages will be 50% more in the year **2035**.

First, find the target sum. A 50% increase on the 2022 sum of **78** is an increase of 78 / 2 = 39. The target sum is 78 + 39 = 117.

The difference between the target sum and the initial sum is 117 - 78 = 39 years.

Each year, the sum of their ages increases by **3** (as each of the three people gets one year older). To find the number of years it will take to increase the sum by 39, divide the required increase by the yearly increase: 39 / 3 = 13 years.

Finally, add these years to the starting year: 2022 + 13 = 2035.

Problem 3:

Note that whether they're wearing the earring in the left ear or the right ear, it's irrelevant to the question. Among the 97% of the women, if half wear two earrings and half none, this is the same as if each wore one.

Therefore, as 3% of the women wear one earring each and the other 97% also wears one earring each on average, the 200 women wear 200 earrings all together.

ANS: 200

Problem 4:

There were **18** coins on the table.

Let **N** be the total number of coins. Initially, there were **N/2** heads, and finally, there were **2N/3** heads. The net increase in the number of heads is 2N/3 - N/2 = N/6.

This change happened because some tails were flipped to heads (t) and some heads were flipped to tails (h). The net change is therefore t - h. We also know that t + h = 5 since five coins were turned over.

This gives us the condition that N/6 = t - h. For this to work with whole numbers, **N/6** must be an odd integer. This leads to three mathematically possible solutions:

- If N/6 = 1, then **N = 6**.
- If N/6 = 3, then **N = 18**.
- If N/6 = 5, then **N = 30**.

All three answers are valid. However, such puzzles often imply a solution that isn't an extreme case (like flipping only tails), making **18** the most common and logical answer.

Problem 5:

Let the two numbers be represented by x and y.

The problem states that their sum is 66, which gives us the equation: x+y=66

The problem also states that their difference is 30, which gives us the equation: x−y=30
We can solve this system of two equations. By adding the first equation to the second, we can eliminate the variable y: (x+y)+(x−y)=66+30 2x=96
Now, we solve for x: x=296 x=48
Substitute the value of x back into the first equation to find y: 48+y=66 y=66−48 y=18
The two numbers are 48 and 18. The question asks for the smaller number.
the correct answer is 18.

Problem 6:
As an old man Marcos' speed is 72+13=773=372+13=773=3 miles per hour, which means he can run 1 mile in 1313 hours, which is 20 minutes. As a younger man, however, his speed was 284+23=28143=6284+23=28143=6 miles per hour. This means he could run 1 mile in 1616 hours, which is 10 minutes. Therefore, it takes him 10 minutes longer to run a mile now, as an older man, compared to when he was a younger man.

ANS: 10

Problem 7:
As Daniel lives 210 kilometers from his parents and drives 72 km/h, it will take him 210 km*72kmh=3512 hours. Converting this to hours and minutes, we get (3512):(60)=175 minutes, which is 2 hours and 55 minutes. If he leaves at 3:10 in the afternoon, he will be there 6:05 in the evening, which when written in 24-hour format, is 18:05.

ANS: 18:05

Problem 8:
Of course. Amna's average speed for the round trip is **3 km/h**.

To get the average speed, you just need to divide the **total distance** she walked by the **total time** it took. The different speeds for uphill and downhill don't complicate the average as long as you use the total values.

- **Total Distance:** 2 km
- **Total Time:** 40 minutes

1. **Total Distance:** Amna walks 1 km to school and 1 km back home, making her total distance **2 km**.
2. **Total Time:** Her trip takes 30 minutes there and 10 minutes back, for a total time of **40 minutes**.
3. **Convert Time to Hours:** The question asks for the speed in kilometers per **hour**, so we need to convert the time.

   $$\frac{40\ minutes}{60\ minutes/hour} = \frac{2}{3}\ hours$$

   Find Average Speed: Now, just divide the total distance by the total time in hours.

   $$2\ km \div \frac{2}{3}\ hours = 3\ km/h$$

   Correct answer: **3 km/h**