Solution:

Problem 1:

The problem states that the number X gives a remainder of (n–1) when divided by n (for n=11,10,...,2). This means $X \equiv (n-1) \pmod n$, which is equivalent to $X \equiv -1 \pmod n$.

So, (X+1) must be perfectly divisible by all numbers from 2 to 11. To find the *smallest* such X, (X+1) must be the Least Common Multiple (LCM) of {2, 3, 4, 5, 6, 7, 8, 9, 10, 11}.

Let's find the LCM:

- Prime factors involved: 2, 3, 5, 7, 11
- Highest powers:
  - 2^3 (from 8)
  - 3^2 (from 9)
  - 5^1 (from 5 or 10)
  - 7^1 (from 7)
  - 11^1 (from 11)

LCM = $2^3 \times 3^2 \times 5 \times 7 \times 11 = 8 \times 9 \times 5 \times 7 \times 11$

LCM = $72 \times 5 \times 7 \times 11 = 360 \times 7 \times 11 = 2520 \times 11 = 27720$.

So, X+1=27720. X=27720–1=27719.

The final answer is **27719**.

Problem 2:

Let the seats be 1, 2, 3, 4, 5. Seat 3 is the middle seat.

- **Rule 1:** Sam and Mike (SM) must sit together.
- **Rule 2:** Adam and Josh (AJ) cannot sit together.

If **Trevor** sits in the middle (seat 3): _ _ T _ _

The SM block must go into seats (1,2) or (4,5).

- If SM are in (1,2): S M T _ _ -- Remaining are Adam and Josh for seats (4,5). They *must* sit next to each other, which breaks Rule 2.
- If SM are in (4,5): _ _ T S M -- Remaining are Adam and Josh for seats (1,2). They *must* sit next to each other, which breaks Rule 2.

Since Adam and Josh will always be forced to sit next to each other if Trevor is in the middle, **Trevor cannot be in the middle seat.**
(For any other friend in the middle seat, there's always a way to place Adam and Josh apart.)
The final answer is **Trevor**.

Problem 3:
Let R(x)=P(x)+Q(x).
Since P(x) has leading coefficient 2 and Q(x) has leading coefficient, the x^2 terms cancel out in R(x).

$$R(x) = (2x^2 + bx + c) + (-2x^2 + ex + f) = (b + e)x + (c + f).$$

So, R(x) is a linear polynomial.

We know:
R(16)=P(16)+Q(16)=54+54=108.
R(20)=P(20)+Q(20)=53+53=106.
For a linear function R(x)=mx+k.
The slope $m = \frac{R(20)-R(16)}{20-16} = \frac{106-108}{4} = \frac{-2}{4} = -\frac{1}{2}$.
So, $R(x) = -\frac{1}{2}x + k$.
Using R(16)=108:
$108 = -\frac{1}{2}(16) + k$.
108=-8+k
k=108+8=116.
So, $R(x) = -\frac{1}{2}x + 116$.
We need to find P(0)+Q(0), which is R(0).

$$R(0) = -\frac{1}{2}(0) + 116 = 116.$$

The final answer is **116**.

Problem 4:

P & Q Location: Both P and Q lie on the horizontal line exactly halfway between the parallel bases (due to angle bisector properties). So, PQ is a horizontal segment.

Coordinates: Let the midpoint of CD be (0,0). So D=(-325,0), C=(325,0), A=(-250,h), B=(250,h).

Find xP: P is equidistant from CD (y=0) and AD. This distance is h/2. Using the distance formula from P(xP,h/2) to line AD, we find xP=−121. (Rejecting outside solution).

Find xQ: Q is equidistant from CD (y=0) and BC. This distance is h/2. Using the distance formula from Q(xQ,h/2) to line BC, we find xQ=121. (Rejecting outside solution).

1. **Calculate PQ:** PQ=|121−(−121)|=242.

The final answer is **242.**

Problem 5:

Condition for "Even" Arrangement: For any pair of identical blocks, one block must be in an odd-numbered position and the other in an even-numbered position. (12 positions: 6 odd, 6 even).

Total Arrengements (N_{total}):

$$N_{total} = \frac{12!}{2!\,2!\,2!\,2!\,2!\,2!} = \frac{12!}{2^6}$$

Number of "Even" Arrengements (N_{even}):

For each of the 6 colors, choose 1 odd position (6 options) and 1 even position (6 options). Repeat for remaining colors:

$$N_{even} = (6 \times 6) \times (5 \times 5) \times (4 \times 4) \times (3 \times 3) \times (2 \times 2) \times (1 \times 1) = (6!)^2$$

Calculate Probability (P=m/n):

$$P = \frac{N_{even}}{N_{total}} = \frac{(6!)^2}{\frac{12!}{2^6}} = \frac{(6!)^2 \times 2^6}{12!} = \frac{16}{231}$$

$$(after\ simplification\ by\ dividing\ by\ common\ factors)$$

Problem 6:

To maximize $x_{76} - x_{16}$ under the given sum constraint, concentrate all negative values on x_1 to x_{16} and all positive values on x_{76} to x_{100}, setting all intermediate terms to zero.

The correct answer is **841**.