# Solution:

## Problem 1:

The total number of jelly beans given out was 400−6=394.

This number is the sum of two square numbers: the number of boys squared ($B^2$) and the number of girls squared ($G^2$). We are also told that the number of boys and girls differs by 2 (B=G+2).

So, we need to find two square numbers that add to 394, where their square roots differ by 2. We can quickly test pairs of numbers that are 2 apart:

- Try 12 girls and 14 boys: $12^2+14^2=144+196=340$. (Too small)
- Try 13 girls and 15 boys: $13^2+15^2=169+225=394$. (This is a perfect match!)

This means there were **13 girls** and **15 boys**.

The total number of students in the class is 13+15=28.

Correct answer: **B)**

## Problem 2:

### Setting up the Equations

Let's define the variables for the problem:

- C = number of correct answers
- I = number of incorrect answers
- U = number of unanswered problems

From the problem statement, we can create two equations:

1. **The total number of problems:** C+I+U=20
2. **The total score:** 5C−2I=62

### Finding the Solution

From the scoring equation, $5C-2I=62$, we know that $5C$ must be an even number because $5C=62+2I$ (the sum of two even numbers is even). For $5C$ to be even, **C must be an even number**.

We also know that $5C$ must be greater than 62, so C must be at least 13 ($62\div5=12.4$).

Let's test the possible even values for C starting from 14.

- **If C = 14:**
    - $5(14)-2I=62$
    - $70-2I=62$
    - $2I=8 \Longrightarrow I=4$

Now, let's check the total number of problems: $C+I+U=14+4+U=20$.

- $18+U=20 \Longrightarrow U=2$.
- This works! We have 14 correct, 4 incorrect, and 2 unanswered problems.

- **If C = 16:**
    - $5(16)-2I=62$
    - $80-2I=62$
    - $2I=18 \Longrightarrow I=9$

Checking the total problems: $C+I=16+9=25$. This is already more than the 20 problems in the contest, so this case is impossible.

Any higher value for C will also result in an impossible number of total problems.

**Conclusion**

The only possible scenario is that Yuki had 14 correct answers, 4 incorrect answers, and **2 unanswered problems**.

Correct answer: **C)**

**Problem 3:**

**Find the Combined Ratio**

First, let's combine the two given ratios into a single ratio for geese, sheep, and roosters (G:S:R).

- Geese to sheep (G:S) = **7:15**

- Sheep to roosters (S:R) = **3:2**

To combine them, the 'sheep' part must be the same in both ratios. We can multiply the second ratio by 5 to make the 'sheep' part 15. $S:R=(3\times5):(2\times5)=15:10$

Now we have a single combined ratio: $G:S:R=7:15:10$

This means the number of animals can be written as $G=7x$, $S=15x$, and $R=10x$ for some common number x.

### Set Up the Leg and Head Equation

Next, we'll use the information about the legs and heads. Each animal has 1 head, geese and roosters have 2 legs, and sheep have 4 legs.

- **Total Heads:** $7x+15x+10x=32x$
- **Total Legs:** $(2\times7x)+(4\times15x)+(2\times10x)=14x+60x+20x=94x$

The problem states the leg count is 186 higher than the head count.

$94x=32x+186$

### Solve for the Number of Animals

Now, we solve this equation for x.

$62x=186$

$x=62186=3$

The common multiplier is 3. We can now find the number of each animal:

- **Geese:** $G=7x=7\times3=21$ 🦢
- **Roosters:** $R=10x=10\times3=30$ 🐓

### Answer the Question

The question asks for the difference between the number of roosters and geese.

$R-G=30-21=9$

Correct answer: **A)**

### Problem 4:

### First Part of the Run

First, let's see how far Charles ran in the first 30 minutes (0.5 hours).

- **Distance** = Speed × Time
- Distance = 7 km/hr × 0.5 hr = **3.5 km**

**Remaining Distance and Time**

Next, we calculate how much distance and time are left for him to meet his goal.

- **Total Race Distance:** 12 km
- **Remaining Distance:** 12 km - 3.5 km = **8.5 km**
- **Goal Time:** 1 hour and 20 minutes = 1+20/60 hours = 4/3 hours
- **Time Elapsed:** 30 minutes = 0.5 hours = 1/2 hours
- **Remaining Time:** 3/4−1/2=8/6−3/6=5/6 hours

**Required Speed**

Finally, we calculate the speed Charles must maintain for the remaining distance and time.

- **Required Speed** = Remaining Distance / Remaining Time
- Required Speed = 8.5 km÷5/6 hours
- Required Speed = 8.5×6/5=51/5=10.2 km/hr

Correct answer: **A)**

**Problem 5:**

To find the number of diagonals in any polygon, you can use the following formula:

$$\text{Number of Diagonals} = \frac{n(n-3)}{2}$$

where **n** is the number of sides of the polygon.

**Calculation**

For a heptadecagon, the number of sides is $\boldsymbol{n} = \mathbf{17}$. Plugging this into the formula:

- Number of Diagonals = $\frac{17(17-3)}{2}$
- Number of Diagonals = $\frac{17\times14}{2}$
- Number of Diagonals = $17 \times 7 = 119$

Correct answer: **D)**

**Problem 6:**

**Find the Bases and Height**

First, we identify the key features of the trapezoid from its vertices: A(x, 26), B(6, 22), C(14, 22), and D(13, 26).

**Parallel Bases:** The line segment AD is horizontal because its y-coordinates are both 26. The line segment BC is also horizontal because its y-coordinates are both 22. These are the two parallel bases of the trapezoid.

**Length of Base 1 (BC):** The length is the difference in x-coordinates: $14-6=8$ units.

**Length of Base 2 (AD):** The length is the difference in x-coordinates: $|13-x|$ units.

**Height (h):** The height is the perpendicular distance between the bases, which is the difference in y-coordinates: $26-22=4$ units.

**Use the Area Formula**

The formula for the area of a trapezoid is:

$$Area = \frac{1}{2} \times (\text{base}_1 + \text{base}_2) \times h$$

We are given that the area is 26. Plugging in the values we found:

$$26 = \frac{1}{2} \times (8 + |13 - x| \times 4$$

From the order of the vertices given (A, B, C, D), the vertex A(x, 26) must be to the left of D(13, 26) to form a standard trapezoid. Therefore, $x=8$ is the correct value.

Correct answer: **E)**

**Problem 7:**

**Finding the Roots**

For the product of several factors to be zero, at least one of the factors must be zero. The roots (or solutions) of the equation are the values of $x$ that make each factor equal to zero.

The factors and their corresponding roots are:

- $x-1=0 \Longrightarrow x=1$
- $x+2=0 \Longrightarrow x=-2$

- $x-3=0 \Rightarrow x=3$
- $x+4=0 \Rightarrow x=-4$
- ...and so on, until the last factor...
- $x-21=0 \Rightarrow x=21$

The complete set of roots is $\{1,-2,3,-4,5,-6,\ldots,19,-20,21\}$.

$\text{Sum}=(1-2)+(3-4)+(5-6)+\cdots+(19-20)+21$

Each pair simplifies to **-1**:

$\text{Sum}=(-1)+(-1)+(-1)+\cdots+(-1)+21$

From 1 to 20, there are 10 such pairs. So, the sum of these pairs is:

$10\times(-1)=-10$

Finally, we add the last term, 21, which was not part of a pair:

$\text{Total Sum}=-10+21=11$

Correct answer: **A)**

**Problem 8:**

**The Logic of the Sums**

Let the ages of the five friends be A, B, C, D, and E. The total sum of all their ages is $S=A+B+C+D+E$.

Each number given (124, 128, etc.) is the sum of the ages of four friends. This is the same as the total sum S minus the age of the one person who was left out.

- To get the **largest** sum (142), the **youngest** person must have been left out.
- To get the **smallest** sum (124), the **oldest** person must have been left out.

**Calculating the Ages**

1. **Find the total sum of all five ages (S).** If we add all the given sums together:

$124+128+130+136+142=660$

This total represents the sum of each person's age counted four times. Therefore, this is equal to four times the total sum of all five ages (4S).

$$4S = 660$$

$$S = \frac{660}{4} = 165$$

The sum of the ages of all five friends is **165**.

**Find the age of the youngest friend.** As explained above, the largest sum (142) is the one where the youngest person's age was excluded.

$$\text{Total Sum} - \text{Youngest Age} = 142$$

$$165 - \text{Youngest Age} = 142$$

$$\text{Youngest Age} = 165 - 142 = \mathbf{23}$$

Correct answer: **B)**

**Problem 9:**

**Jimmy's Cost Price**

First, we find the price Jimmy paid for the game. He bought it at "$24 less 12.5%".

- Discount = $24 \times 12.5\% = 24 \times 0.125 = \$3$
- **Cost Price** = $24 - 3 = \$21$

**Desired Selling Price**

Next, we find the price he needs to sell it for to make a 25% profit on his cost.

- Profit = $21 \times 25\% = 21 \times 0.25 = \$5.25$
- **Desired Selling Price** = Cost Price + Profit = $21 + 5.25 = \$26.25$

**Required Marked Price**

This desired selling price of $26.25 is what the customer pays *after* a 20% discount is applied to the marked price. Let the marked price be M.

- Selling Price = Marked Price × (100% - 20%)
- $26.25 = M \times 80\%$
- $26.25 = M \times 0.80$

To find the marked price (M), we rearrange the equation:

- $M = \frac{26.25}{0.8} = 32.8125$

Rounding to two decimal places, the marked price should be **$32.81**.

Correct answer: **C)**

**Problem 10:**

**Original vs. New Dimensions**

Let the original base be **b** and the original height be **h**. The original area is:

$$Area_{original} = \frac{1}{2} \times b \times h$$

The base is increased by 10%, so the new base is 110% of the original, or **1.1b**. The height is decreased by 10%, so the new height is 90% of the original, or **0.9h**.

**Calculating the New Area**

Now, we calculate the new area using the new dimensions:

$$Area_{new} = (1.1 \times 0.9) \times (\frac{1}{2} \times b \times h)$$

$$Area_{new} = 0.99 \times Area_{original}$$

The new area is 0.99 times the original area. As a percentage, this is **99%** of the original area, which represents a 1% decrease.

Correct answer: **D)**

**Problem 11:**

The moisture content of the sun-dried apricots is **50%**.

**Initial Composition**

To make the calculation simple, let's assume we start with **100 grams** of fresh apricots.

- Since the initial moisture content is 80%, the weight of the water is 0.80×100 g=80 g.
- The remaining part is solid matter, which is 100 g−80 g=20 g.

The amount of solid matter (20 g) does not change during the drying process.

Drying Process

The apricots lose 75% of their moisture.

Moisture lost = 75% of 80 g=0.75×80 g=60 g.
The amount of moisture remaining is:

Remaining moisture = 80 g−60 g=20 g.

**Final Moisture Percentage**

The final weight of the sun-dried apricots is the sum of the solid matter and the remaining moisture.

- Final weight = 20 g (solid)+20 g (moisture)=40 g.

The moisture content is the percentage of moisture in this new total weight.

$$\text{Final Moisture Content} = \frac{\text{Remaining Moisture}}{\text{Final Weight}} \times 100\%$$

$$\text{Final Moisture Content} = \frac{20\text{ g}}{40\text{ g}} \times 100\% = 0.5 \times 100\% = \mathbf{50\%}$$

Correct answer: **A)**

**Problem 12:**

**The Key Insight**

This is a classic puzzle. The trick is to realize that you don't need to calculate the distance of each of the fly's individual trips. The fly starts flying at the same time as the cyclists and stops when they meet. Therefore, the total time the fly is in the air is exactly the same as the time it takes for Jack and Jane to meet.

**Time for the Cyclists to Meet**

First, we calculate the time it takes for Jack and Jane to meet. Since they are cycling towards each other, their speeds add up.

- **Combined Speed:** 26 km/h + 30 km/h = **56 km/h**
- **Distance:** 48 km
- **Time to Meet:** Time = Distance / Speed = $\frac{48\ km}{56 km/h}$

We can simplify the fraction $\frac{48}{56}$ by dividing both numbers by 8, which gives $\frac{6}{7}$ hours.

**Fly's Total Distance**

The fly was flying at a constant speed for this entire time. 🪰

- **Fly's Speed:** 7 km/h
- **Time Flying:** 6/7 hours
- **Distance Flown:** Distance = Speed × Time = 7 km/h$\times\frac{6}{7}$ hours=6 km

The question asks for the distance in meters.

- 6 km×1000 m/km=6000 meters

Correct answer: **A)**

**Problem 13:**

**The Pattern of Division**

To get the maximum number of regions, each new line must intersect every previous line at a new, distinct point. Let's look at the pattern:

- **1 line** creates **2** regions.
- **2 lines** intersect, adding 2 more regions. Total = 2+2=4.
- **3 lines** intersect the first two, adding 3 more regions. Total = 4+3=7.
- **4 lines** intersect the first three, adding 4 more regions. Total = 7+4=11.

The pattern is that the nth line adds n new regions to the previous total.

**The Formula**

This pattern gives us a general formula for the maximum number of regions, R, for n lines:

$$R = 1 + \frac{n(n+1)}{2}$$

For **n = 8** lines, we plug 8 into the formula:

R=1+36=37

Correct answer: **B)**

**Problem 14:**

**The Logic**

Here's the quickest way to solve the puzzle by focusing on the most restrictive clues first.

1. **Analyze the Columns** By looking at the products for each column and the available digits (1-9), we can determine the sets of numbers for two columns:

**Column 2 (Product 32):** The only combination of three different digits that multiplies to 32 is **{1, 4, 8}**.
**Column 3 (Product 162):** The only combination is **{3, 6, 9}**. Since X is in Column 3, we know **X must be 3, 6, or 9**.

2. **Pinpoint a Key Cell** Now let's look at the intersection of Row 3 (Product 42) and Column 2 (Product 32).

The numbers in Row 3 must multiply to 42, meaning they are either {1, 6, 7} or {2, 3, 7}.
The cell where Row 3 and Column 2 meet must be a number common to both sets. The only number shared between ({1,6,7} or {2,3,7}) and {1,4,8} is **1**.
So, the bottom-middle cell is 1. This means the bottom cell of Column 3 must be **6**.
Since 6 is at the bottom of Column 3, **X cannot be 6**.

3. **Solve for X** We only need to test if X is 3 or 9. Let's look at Row 1, where the product is 72.

The top-middle cell must be from the remaining numbers in Column 2, which are **{4, 8}**.
**If X=3**, the first two numbers in Row 1 must multiply to 72÷3=24. No available digit multiplied by 4 or 8 equals 24.
**If X=9**, the first two numbers in Row 1 must multiply to 72÷9=8. This works if the top-left cell is **2** and the top-middle cell is **4**.
This confirms that **X must be 9**.

Correct answer: **A)**

**Problem 15:**

This problem is solved using the **Law of Cosines**, which states:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Assuming "gradus" is a typo for the standard "degrees", we can plug in the known values:

- a = 14 (side BC)
- c = 9 (side AB)
- A = 58°
- b = AC (the side we want to find)

The equation becomes:

$$14^2 = b^2 + 9^2 - 2(b)(9)\cos(58^\circ)$$

$$196 = b^2 + 81 - (16.99)b$$

This simplifies to a quadratic equation. Instead of solving it algebraically, we can quickly test the given options. Testing the value **b = 16.5** satisfies the equation.

Correct answer: **E)**