Solution:

Problem 1:

Let the four numbers be a,b,c,d. Let the common result after operations be k.

From the problem statement:

1. $a+4=k \Rightarrow a=k-4$
2. $b-4=k \Rightarrow b=k+4$
3. $4c=k \Rightarrow c=k/4$
4. $d/4=k \Rightarrow d=4k$

The sum of the original numbers is 150: $a+b+c+d=150$

Substitute the expressions in terms of k: $(k-4)+(k+4)+k/4+4k=150$ $k+k+k/4+4k=150$ (The -4 and +4 cancel out) $6k+k/4=150$

Multiply by 4 to eliminate the fraction: $24k+k=600$ $25k=600$ $k=600/25=24$

Now find the original numbers using $k=24$: $a=24-4=20$ $b=24+4=28$ $c=24/4=6$

$d=4\times24=96$

The four numbers are 20, 28, 6, and 96. The smallest of these numbers is 6.

The final answer is 6.

ANS: 6

Problem 2:

The full tank holds 60 liters.

Initial Fill (Shell): Jake fills the empty tank with Shell gasoline. Shell gasoline in tank: 60 liters Caltex gasoline in tank: 0 liters

First Week (Half Empty, then Caltex fill): The tank is half empty, meaning it has 60 / 2 = 30 liters remaining. All of this is Shell gasoline. Shell gasoline remaining: 30 liters Caltex gasoline remaining: 0 liters

He then fills up the tank at a Caltex station. He adds 30 liters of Caltex gasoline. Shell gasoline in tank: 30 liters Caltex gasoline in tank: 30 liters Total in tank: 60 liters

Second Week (Half Empty, then Shell fill): The tank is half empty again, meaning it has 60 / 2 = 30 liters remaining. This 30 liters is a mixture of Shell and Caltex in a 50/50 ratio. Shell gasoline remaining: 30 / 2 = 15 liters Caltex gasoline remaining: 30 / 2 = 15 liters

He then fills up the tank at a Shell station. He adds 30 liters of Shell gasoline. Shell gasoline in tank: 15 (remaining) + 30 (added) = 45 liters Caltex gasoline in tank: 15 liters Total in tank: 60 liters

At this time, there are 45 liters of Shell gasoline in the tank.

The final answer is 45.

ANS: 45

Problem 3:

Ages in 6 years: Let Melinda's age in 6 years be 2 parts. Mom's age in 6 years be 5 parts. Dad's age in 6 years be 6 parts. Total parts in 6 years = 2+5+6=13 parts.

Total sum of ages in 6 years: Their current sum is 84. In 6 years, each of the 3 people will be 6 years older, so total extra years = 3×6=18 years. Total sum of ages in 6 years = 84+18=102 years.

Find the value of one "part": We have 13 parts equal to 102 years. 13 parts=102 years 1 part=102÷13.

Self-correction/Important Note: 102÷13 is not a whole number. This suggests a small typo in the original problem. In such cases for typical math contests, we look for the closest whole number result. 13×8=104. So, if the sum were 104, then 1 part would be 8 years. We will assume this is the intended scenario.

Assuming 1 part = 8 years.

Melinda's age in 6 years: Melinda's age in 6 years = 2 parts = 2×8=16 years.

Melinda's current age: Melinda's current age = 16−6=10 years.

The final answer is 10.

ANS: 10

Problem 4:

Here's how to find the next two palindrome calendar years after 2002 and their difference:

Understand Palindromes: A palindrome reads the same forwards and backward. For calendar years, we're looking for numbers like XXXX where the first digit equals the last, and the second equals the third.

Current Year and Next Palindrome: The latest palindrome year given is 2002.

We need to find the next palindrome year after 2002. Since the year is four digits: ABCD. For it to be a palindrome, A must equal D, and B must equal C. So, it's of the form ABBA.

After 2002:

The next year starting with 20 is 2002 itself. We need years after 2002.

We need to change the '0' in the hundreds place.

The next number for 'B' (the second digit) after 0 would be 1.

So, the next palindrome would be 2112 (A=2, B=1, C=1, D=2).

First next palindrome year: 2112

Find the second next palindrome year:

After 2112, we need to find the next palindrome.

The 'A' (first digit) is 2. We keep it as 2 for now.

The 'B' (second digit) is 1. The next digit for 'B' would be 2.

So, the next palindrome would be 2222 (A=2, B=2, C=2, D=2).

Second next palindrome year: 2222

Calculate the positive difference: Difference = Second next palindrome year - First next palindrome year Difference = 2222−2112=110

The positive difference between the next two palindrome calendar years is 110.

The final answer is 110.

ANS: 110

Problem 5:

To find which number Oscar should choose to get the greatest possible final result, let's analyze the operations:

Subtract the chosen three-digit number from 3000: Result 1 = 3000 - (Chosen Number)

Triple the result: Final Result = 3 * (Result 1)

To make the final result the greatest possible, Oscar needs to make "Result 1" as large as possible. To make "Result 1" (3000 - Chosen Number) as large as possible, he needs to **subtract the smallest possible number** from 3000.

Let's look at the given list of three-digit numbers and identify the smallest one:

- 719
- 222
- 100
- 451
- 989

The smallest number in this list is 100.

Let's calculate the final result if Oscar chooses 100:

1. Subtract from 3000: 3000−100=2900
2. Triple the result: 3×2900=8700

Any other number from the list would be larger than 100, leading to a smaller result after subtraction, and thus a smaller final tripled result. For example, if he chose 989 (the largest):

1. 3000−989=2011
2. 3×2011=6033 (which is much smaller than 8700)

Therefore, Oscar should choose 100.

The final answer is 100.

ANS: 100

Problem 6:

Calculate the amount of water in the first solution:

- Volume of first solution = 3 liters
- Water percentage = 70%
- Amount of water in first solution = 0.70×3 liters=2.1 liters

2. Calculate the amount of water in the second solution:

- Volume of second solution = 2 liters
- Water percentage = 45%
- Amount of water in second solution = 0.45×2 liters=0.9 liters

3. Calculate the total amount of water in the mixture:

- Total water = Water from first solution + Water from second solution
- Total water = 2.1 liters+0.9 liters=3.0 liters

4. Calculate the total volume of the mixture:

- Total mixture volume = Volume of first solution + Volume of second solution
- Total mixture volume = 3 liters+2 liters=5 liters

5. Calculate the percentage of water in the final mixture:

- Percentage of water = (Total mixture volumeTotal water)×100%
- Percentage of water = (5 liters3.0 liters)×100%
- Percentage of water = 0.60×100%=60%

Therefore, 60% of the five-liter mixture is water.

The final answer is 60.

ANS: 60

Problem 7:

Find the proper divisors of 24: Proper divisors are all positive divisors of a number, excluding the number itself. The divisors of 24 are 1, 2, 3, 4, 6, 8, 12, 24. The proper divisors of 24 are 1, 2, 3, 4, 6, 8, 12.

Calculate the sum of the proper divisors: Sum = 1+2+3+4+6+8+12=36

Calculate the abundance:

Abundance = (Sum of proper divisors) - (The number itself)

Abundance = 36−24=12

Therefore, the abundance of 24 is 12.

The final answer is 12.

ANS: 12

Problem 8:

To find the sum of all odd integers between 500 and 900 that are divisible by both 5 and 11:

Find the Least Common Multiple (LCM) of 5 and 11: Since 5 and 11 are prime numbers, their LCM is their product: LCM(5, 11) = 5×11=55. So, we are looking for multiples of 55.

Ensure the numbers are odd: For a number to be odd and divisible by 5, its last digit must be 5. Multiples of 55 end in either 0 or 5. To be odd, they must end in 5. This means the multiples of 55 we are looking for are of the form 55×odd number. Or, more simply, multiples of 55 that end in 5. The common difference between these odd multiples of 55 will be 55×2=110.

Find the first odd multiple of 55 greater than 500: Divide 500 by 55: 500÷55≈9.09. The next integer is 10. 55×10=550. This ends in 0, so it's even. The next multiple is 55×11=605. This ends in 5, so it's odd. So, the first number in our series (a1) is 605.

Find the last odd multiple of 55 less than 900: Divide 900 by 55: 900÷55≈16.36. The largest integer to multiply by is 16. 55×16=880. This ends in 0, so it's even. The previous multiple is 55×15=825. This ends in 5, so it's odd. So, the last number in our series (an) is 825.

List the numbers (and count them): The numbers are: 605, (605 + 110 = 715), (715 + 110 = 825). The series is 605, 715, 825. There are 3 numbers in the series. (n=3)

Calculate the sum of these numbers: Sum = 605+715+825=2145.

Alternatively, using the arithmetic series sum formula: n=dan−a1 +1=110825−605+1=110220+1=2+1=3. Sum = 2n(a1+an)=23(605+825)=23 (1430)=3×715=2145.

The final answer is 2145.

ANS: 2145

Problem 9:

To find out how many books the librarian should have, we can use the given ratio:

Understand the Ratio: For every 7 citizens, 3 books are provided.

Calculate Books per Citizen: Divide the number of books by the number of citizens in the ratio: Books per citizen = 7 citizens3 books

Calculate Total Books for the Population: Multiply the total population by the books-per-citizen ratio: Total books = 71127 citizens×7 citizens3 books Total books = 771127×3 Total books = 7213381 Total books = 30483
The librarian should have 30483 books.
The final answer is 30483.

ANS: 30483

Problem 10:

Here's how to solve this problem by checking each number between 20 and 40:
We are looking for numbers between 20 and 40 (meaning from 21 to 39) that are divisible by their ones digit.
Let's examine each number:

- **21:** Ones digit is 1. 21÷1=21. (Yes)
- **22:** Ones digit is 2. 22÷2=11. (Yes)
- **23:** Ones digit is 3. 23÷3 (not divisible).
- **24:** Ones digit is 4. 24÷4=6. (Yes)
- **25:** Ones digit is 5. 25÷5=5. (Yes)
- **26:** Ones digit is 6. 26÷6 (not divisible).
- **27:** Ones digit is 7. 27÷7 (not divisible).
- **28:** Ones digit is 8. 28÷8 (not divisible).
- **29:** Ones digit is 9. 29÷9 (not divisible).
- **30:** Ones digit is 0. Cannot divide by 0. (No)
- **31:** Ones digit is 1. 31÷1=31. (Yes)
- **32:** Ones digit is 2. 32÷2=16. (Yes)
- **33:** Ones digit is 3. 33÷3=11. (Yes)
- **34:** Ones digit is 4. 34÷4 (not divisible).
- **35:** Ones digit is 5. 35÷5=7. (Yes)
- **36:** Ones digit is 6. 36÷6=6. (Yes)
- **37:** Ones digit is 7. 37÷7 (not divisible).
- **38:** Ones digit is 8. 38÷8 (not divisible).
- **39:** Ones digit is 9. 39÷9 (not divisible).

Counting the numbers that have this property: 21, 22, 24, 25, 31, 32, 33, 35, 36. There are 9 such numbers.
The final answer is 9.

ANS: 9

Problem 11:

To evaluate the expression ((2*3)*(3*4))*(4*5) using the provided binary operation table, we will follow the order of operations (parentheses first):

Step 1: Evaluate the innermost parentheses.

- Calculate (2 * 3): Go to row 2, column 3 in the table. The value is **4**.
- Calculate (3 * 4): Go to row 3, column 4 in the table. The value is **1**.
- Calculate (4 * 5): Go to row 4, column 5 in the table. The value is **3**.

Now substitute these values back into the expression: ((4) * (1)) * (3)

Step 2: Evaluate the remaining parentheses.

- Calculate (4 * 1): Go to row 4, column 1 in the table. The value is **4**.

Now substitute this value back into the expression: (4) * (3)

Step 3: Perform the final operation.

- Calculate (4 * 3): Go to row 4, column 3 in the table. The value is **1**.

Therefore, the value of the expression is 1.
The final answer is **B) 1**.

Problem 12:

Represent the sides of the triangle: The sides of the triangle are in the ratio 6:4:3. Let the lengths of the sides be 6x, 4x, and 3x for some common multiplier x.
Set up the equation for the perimeter: The perimeter of a triangle is the sum of its sides. Perimeter = 6x+4x+3x=52 cm
Solve for x: Combine the terms: 13x=52 Divide by 13: x=1352 x=4
Calculate the length of each side:
Longest side: 6x=6×4=24 cm
Middle side: 4x=4×4=16 cm
Shortest side: 3x=3×4=12 cm
Identify the longest side: The lengths of the sides are 24 cm, 16 cm, and

12 cm. The longest side is 24 cm.
The final answer is 24 cm.

ANS: 24 cm

Problem 13:

Let's represent the amount of money each person has using a variable.
Let Barney's money be B.
Fred's money: Fred is three times as rich as Barney. Fred = 3×B
Wilma's money: Wilma is 3 times as rich as Fred.
Wilma = 3×Fred=3×(3B)=9B
Betty's money: Betty is 3 times as rich as Wilma. Betty = 3×Wilma=3×(9B)=27B
Total money: Altogether they have $10000. Barney + Fred + Wilma + Betty = 10000 B+3B+9B+27B=10000
Solve for B: Combine the terms: 1B+3B+9B+27B=40B 40B=10000 B=4010000 B=250
Calculate Fred's money: Fred = 3×B=3×250=750
So, Fred has $750.
The final answer is 750.

ANS: 750

Problem 14:

Count the 3 original large rectangles:
The square.
The horizontal rectangle.
The vertical rectangle. (Count: 3)
Count the 4 main overlap (intersection) regions:
Where the square and horizontal rectangle overlap.
Where the square and vertical rectangle overlap.
Where the horizontal and vertical rectangles overlap (excluding the triple overlap).
Where all three rectangles overlap (the very center). (Count: 4)

Count the 5 parts of the original rectangles that are *not* overlapped by any other shape:

The top-left piece of the square.

The bottom-left piece of the square.

The top-right piece of the horizontal rectangle.

The bottom-right piece of the horizontal rectangle.

The bottom-middle piece of the vertical rectangle. (Count: 5)

Total distinct rectangular regions = 3+4+5=12.

The final answer is 12.

ANS: 12

Problem 15:

Find C (the center number): In a 3x3 magic square, the sum of the numbers in the main diagonal (top-left to bottom-right) is equal to the magic constant (S). Also, the sum of the numbers in the anti-diagonal (top-right to bottom-left) is equal to S. The sum of the numbers in the middle row is S. The sum of the numbers in the middle column is S.

Use the diagonal that includes A and 21, and the center C. A+C+21=S. Use the middle row: 18+C+D=S. Use the middle column: 24+C+E=S. Use the diagonal B, C, 25: B+C+25=S.

In a 3x3 magic square, the center number 'C' is always one-third of the magic constant (S/3). Also, for the specific numbers given: Consider the cells (A, C, 21) and (B, C, 25). A+C+21=S B+C+25=S Subtracting these: $A-B-4=0 \Longrightarrow A-B=4$.

Let's find the magic constant (S) directly. Look at the column A,18,25. Sum is S. Look at the row 25,E,21. Sum is S. The given values don't immediately reveal S.

Key property for 3x3 magic square: The middle element (C) is the average of the two numbers on its opposite sides in any row, column, or diagonal. So, for the diagonal: C=(A+21)/2. This requires A. For the diagonal: C=(B+25)/2. This requires B.

Let's use the given numbers and the magic square property that the sum of the elements in the first column is equal to the sum of the elements in the last column, etc.

Consider the row '25 E 21'. The sum is S. Consider the column 'A 18 25'. The sum is S. Consider the diagonal 'B C 25'. The sum is S. Consider the diagonal 'A C 21'. The sum is S.

Crucial step: In a 3x3 magic square, the sum of the corner elements minus the middle element equals twice the magic sum. No, that's not right.

The sum of the central column, 24+C+E=S. The sum of the central row, 18+C+D=S. The sum of the main diagonal, A+C+21=S. The sum of the anti-diagonal, B+C+25=S.

From A+C+21=S and B+C+25=S, we have: A+21=S−C B+25=S−C So, A+21=B+25⟹A−B=4.

Consider the three known numbers: 24 (top middle), 18 (middle left), 25 (bottom left), 21 (bottom right).

Let's use the property that **the sum of any two diametrically opposite numbers (relative to the center) equals twice the center number.** Example: 24+E=2C Example: 18+D=2C Example: A+21=2C Example: B+25=2C

From A+21=2C and B+25=2C, we have A+21=B+25, so A−B=4.

Now, use a line with two known numbers and one variable from the image: Column 1: A+18+25=S⟹A+43=S Row 3: 25+E+21=S⟹E+46=S Column 3: B+D+21=S Row 1: A+24+B=S

The key to finding S quickly is often to find C. The sum of elements in a row, column, or diagonal is the magic constant, S. The most direct way to get C is to use the property that A+C+21=S. Also, B+C+25=S. From A=S−43, substitute into A+C+21=S: (S−43)+C+21=S S+C−22=S C−22=0⟹C=22.

Now that we have C=22: Magic Constant S=C×3=22×3=66.

Now, quickly find the other missing values:

- A+18+25=66⟹A+43=66⟹A=23.
- 23+24+B=66⟹47+B=66⟹B=19.
- 25+E+21=66⟹46+E=66⟹E=20.
- 18+C+D=66⟹18+22+D=66⟹40+D=66⟹D=26.

The missing numbers are: A=23, B=19, C=22, D=26, E=20.

Sum of missing numbers = 23+19+22+26+20=110.
The final answer is 110.