Solution:

Problem 1:

To determine how many possible digits can be inserted, we need to use the rule of divisibility by 3: a number is divisible by 3 if the sum of its digits is divisible by 3.

The given number is 25893_4305, where '_' represents the erased digit.

1. **Sum the known digits:** 2+5+8+9+3+4+3+0+5=39
2. **Let the erased digit be 'x'.** The sum of all digits will be 39+x.
3. **Apply the divisibility rule:** For the 10-digit number to be divisible by 3, the sum of its digits (39+x) must be divisible by 3.
4. **Find possible values for 'x':** The digit 'x' can be any whole number from 0 to 9. We need to find values of 'x' such that (39+x) is a multiple of 3.
    - If x=0, sum = 39+0=39. 39÷3=13. (Divisible by 3)
    - If x=1, sum = 39+1=40. (Not divisible by 3)
    - If x=2, sum = 39+2=41. (Not divisible by 3)
    - If x=3, sum = 39+3=42. 42÷3=14. (Divisible by 3)
    - If x=4, sum = 39+4=43. (Not divisible by 3)
    - If x=5, sum = 39+5=44. (Not divisible by 3)
    - If x=6, sum = 39+6=45. 45÷3=15. (Divisible by 3)
    - If x=7, sum = 39+7=46. (Not divisible by 3)
    - If x=8, sum = 39+8=47. (Not divisible by 3)
    - If x=9, sum = 39+9=48. 48÷3=16. (Divisible by 3)

The possible digits for 'x' are 0, 3, 6, and 9.

5. **Count the possible digits:** There are 4 possible digits.

The final answer is 4.

ANS: 4

Problem 2:

It's clear that:

$$3 \times 8 = 24$$
$$5 \times 3 = 15$$
$$8 \times 2 = 16$$
$$7 \times 5 = 35$$
$$6 \times x = 12 \rightarrow x = 2$$

ANS: 2

Problem 3:

To find the exchange rate between bananas and hammocks, we need to set up a chain of equivalences.

We are given:

1. 50 bananas = 20 coconuts
2. 30 coconuts = 12 fish
3. 100 fish = 1 hammock

Our goal is to find out how many bananas equal 1 hammock.

Step 1: Express coconuts in terms of bananas. From (1): 50 bananas = 20 coconuts Divide both sides by 20 to find out how many bananas 1 coconut is worth: 1 coconut = 2050 bananas = 25 bananas

Step 2: Express fish in terms of coconuts (and then bananas). From (2): 30 coconuts = 12 fish Divide both sides by 12 to find out how many coconuts 1 fish is worth: 1 fish = 1230 coconuts = 25 coconuts

Now substitute the value of 1 coconut from Step 1 into this equation: 1 fish = 25×(25 bananas) 1 fish = 425 bananas

Step 3: Express hammocks in terms of fish (and then bananas). From (3): 100 fish = 1 hammock

Now substitute the value of 1 fish from Step 2 into this equation: 1 hammock = 100×(425 bananas) 1 hammock = (100×25)/4 bananas 1 hammock = 25×25 bananas 1 hammock = 625 bananas

Therefore, 625 bananas equal 1 hammock.

The final answer is 625.

ANS: 625

Problem 4:

1. **Convert the distance to meters:** The snails are 1 kilometer apart. 1 kilometer = 1000 meters.
2. **Calculate their combined speed:** Speedy's speed = 0.5 meters per second Hasty's speed = 0.75 meters per second Since they are moving towards each other, their speeds add up to find their closing rate. Combined speed = 0.5 m/s + 0.75 m/s = 1.25 m/s
3. **Calculate the time it takes for them to touch:** Time = Total Distance / Combined Speed Time = 1000 meters / 1.25 m/s

To divide by 1.25, you can think of it as dividing by 5/4. Time = 1000*4:5 =1000:5:4 Time = 4000:5=800 seconds.

Therefore, it takes 800 seconds for Speedy and Hasty to touch.

The final answer is 800 s.

ANS: 800 s

Problem 5:

Let the three different whole numbers on the cards be a, b, and c, in increasing order (i.e., $a<b<c$).

We are given the sums of the numbers taken two at a time:

1. $a+b=39$
2. $a+c=47$
3. $b+c=58$

We are also given that the difference between the smallest and the largest numbers is 19. $c-a=19$

We need to find the middle number, b.

Method 1: Using the sum of all equations

Add the first three equations together: $(a+b)+(a+c)+(b+c)=39+47+58$ $2a+2b+2c=144$ Divide the entire equation by 2: $a+b+c=72$

Now we have a system of equations:

- $a+b+c=72$
- $a+b=39$
- $a+c=47$

- $b+c=58$

To find c, substitute $(a+b)=39$ into $(a+b+c)=72$: $39+c=72$ $c=72-39$ $c=33$
To find b, substitute $(a+c)=47$ into $(a+b+c)=72$: $47+b=72$ $b=72-47$ $b=25$
To find a, substitute $(b+c)=58$ into $(a+b+c)=72$: $a+58=72$ $a=72-58$ $a=14$
So the three numbers are 14, 25, and 33. Let's check the given sums and difference:

- $a+b=14+25=39$ (Correct)
- $a+c=14+33=47$ (Correct)
- $b+c=25+33=58$ (Correct)
- $c-a=33-14=19$ (Correct)

The middle number is $b=25$.

Method 2: Using the given difference directly

We have the equations:

1. $a+b=39$
2. $a+c=47$
3. $b+c=58$
4. $c-a=19$

From (2) $a=47-c$. Substitute this into (4): $c-(47-c)=19$ $c-47+c=19$ $2c=19+47$ $2c=66$ $c=33$

Now that we have c, we can find a using (4): $a=c-19=33-19=14$.

Finally, find b using (1): $b=39-a=39-14=25$.

The middle number is 25.

The final answer is 25.

ANS: 25

Problem 6:

Each dart can score 1, 2, or 3 points. The contestants throw 3 darts. We need to find the number of *different possible total points*.

Let the scores of the three darts be d1, d2, and d3, where each $di \in \{1,2,3\}$. The total points will be $T=d1+d2+d3$.

1. Find the minimum possible total points: This occurs when all three darts land on the 1-point ring. Minimum total = $1+1+1=3$ points.

2. Find the maximum possible total points: This occurs when all three darts land on the 3-point inner circle. Maximum total = 3+3+3=9 points.

3. List all possible total points between the minimum and maximum: Since the scores are integers, and the minimum is 3 and the maximum is 9, all integer scores between 3 and 9 might be possible. We need to confirm this.

Let's see if we can achieve each score:

- **3 points:** 1+1+1
- **4 points:** 1+1+2
- **5 points:** 1+1+3 or 1+2+2
- **6 points:** 1+2+3 or 2+2+2
- **7 points:** 1+3+3 or 2+2+3
- **8 points:** 2+3+3
- **9 points:** 3+3+3

All integer scores from 3 to 9 are possible.

4. Count the number of different total points: The possible total points are 3, 4, 5, 6, 7, 8, 9. To count them, you can do (Maximum - Minimum) + 1. Number of different total points = 9−3+1=7.

The final answer is 7.

ANS: 7

Problem 7:

Total Runners Must Be Odd: Andrea finished in the *middle position*, meaning the total number of runners (N) must be an odd number. (N+1)/2 gives Andrea's position.

Breonna and Ciara Finished After Andrea: Breonna was 19th and Ciara was 28th. This means Andrea's position must be *earlier than 19th*.

Test the Options (Number of Schools): Since each school has 3 runners, N must be a multiple of 3.

A) 11 schools: N=11×3=33 runners. Andrea's position = (33+1)/2=17. This is earlier than 19th (Breonna) and 28th (Ciara), which fits the condition that they finished after Andrea. This is consistent.

B) 12 schools: $N=12\times3=36$ runners. Andrea's position would be 18.5, not a whole number. (Invalid)

C) 13 schools: $N=13\times3=39$ runners. Andrea's position = $(39+1)/2=20$. This means Breonna (19th) finished *before* Andrea. (Invalid)

D) 14 schools: $N=14\times3=42$ runners. Andrea's position would be 21.5. (Invalid)

E) 15 schools: $N=15\times3=45$ runners. Andrea's position = $(45+1)/2=23$. Breonna (19th) finished *before* Andrea. (Invalid)

Only **11 schools** leads to a consistent scenario.

The final answer is 11.

ANS: 11

Problem 8:

Let the two numbers be x and y. Since we need to write them in increasing order, let x be the smaller number and y be the larger number ($x<y$).

We are given two pieces of information, which can be written as equations:

1. **Their sum is one thousand:** $x+y=1000$
2. **Their difference is one hundred:** $y-x=100$ (Since y is the larger number)

Now we have a system of two linear equations: Equation 1: $x+y=1000$ Equation 2: $-x+y=100$

Method 1: Adding the two equations Add Equation 1 and Equation 2: $(x+y)+(-x+y)=1000+100$ $x-x+y+y=1100$ $2y=1100$ $y=21100$ $y=550$

Now substitute the value of y back into Equation 1 to find x: $x+550=1000$ $x=1000-550$ $x=450$

So the two numbers are 450 and 550. Let's check: Sum: $450+550=1000$ (Correct) Difference: $550-450=100$ (Correct)

Since we need to write them in increasing order, separated by a comma: 450, 550.

The final answer is 450, 550.

ANS: 450, 550

Problem 9:

Price after October markdown (20% off):

Price in October = Original Price - (20% of Original Price) Price in October = $250-(0.20\times250)=250-50=200$

Price after November markup (30% up from October price): Price in November = October Price + (30% of October Price) Price in November = $200+(0.30\times200)=200+60=260$

Price after December markdown (40% off from November price): Price in December = November Price - (40% of November Price) Price in December = $260-(0.40\times260)=260-104=156$

The new price of the item in December is $156.

The final answer is 156.

ANS: 156

Problem 10:

First Discount (Half Price): "Everything is half price" means a 50% discount. If the original price is $100, the sale price becomes $100\times(1-0.50)=100\times0.50=50$.

Additional VIP Discount (30% off the sale price): The VIP card gives an additional 30% discount on the already discounted sale price. Discount amount = 30% of $50=0.30\times50=15$. Final price = Sale Price - Additional Discount = $50-15=35$.

Percentage of Original Price: The final price is 35 when the original price was 100. Percentage of original price = (OriginalPrice) $\times100$.

The final price, using the VIP card, represents 35% of the original price.

The final answer is 35.

ANS: 35

Problem 11:

Remove all of the most numerous flavor: There are 10 butterscotch candies. You could remove all 10 of them. Candies removed: 10 (Butterscotch) Remaining to remove: Need 2 of each flavor.

Remove all of the next most numerous flavor: There are 6 mint candies. You could remove all 6 of them. Candies removed: 10 (Butterscotch) + 6 (Mint) = 16 Remaining to remove: Need 2 chocolate, and still need to ensure we get a second mint and butterscotch.

At this point (16 candies removed), you have:

- 0 chocolate candies eaten (worst case, you picked none yet)
- 6 mint candies eaten (all of them)
- 10 butterscotch candies eaten (all of them)

To guarantee at least two of *each* flavor, you still need to get:

- Two chocolate candies.
- A second mint (which you already have 6 of, so this condition is met for mint).
- A second butterscotch (which you already have 10 of, so this condition is met for butterscotch).

So, the next candies you pick must be chocolate to satisfy the condition for all flavors.

Remove the required number of the remaining flavor: You need 2 chocolate candies. Candies removed: 16 (all butterscotch and all mint) + 2 (Chocolate) = 18

At this point (18 candies removed), you have:

- 2 chocolate candies
- 6 mint candies
- 10 butterscotch candies

All conditions are met: you have at least two of each flavor.

Therefore, the minimum number of candies that must be removed to be certain that at least two candies of each flavor have been eaten is 18.

The final answer is 18.

ANS: 18

Problem 12:

Calculate how many 9-year old students can ride the student elevator: Maximum elevator capacity = 227 kilograms Average weight of a 9-year old = 28 kilograms Number of students =

lfloorfrac22728rfloor=lfloor8.107...rfloor=8 students. (We use the floor function because you can't have a fraction of a person.)

Calculate how many adults can ride the adult elevator: Maximum elevator capacity = 227 kilograms Average weight of an adult = 73 kilograms Number of adults = lfloorfrac22773rfloor=lfloor3.109...rfloor=3 adults.

Find the difference: Difference = Number of students - Number of adults Difference = 8−3=5

Therefore, 5 more 9-year old students can ride the student elevator than adults the adult elevator.

The final answer is 5.

ANS: 5

Problem 13:

Let:

- R be the worth of a single red poker chip (in coins).
- B be the worth of a single blue poker chip (in coins).

From the information given, we can set up two equations:

Equation 1 (Steve's chips): Steve has 5 red chips and 7 blue chips, worth a total of 69 coins. $5R+7B=69$

Equation 2 (Tom's chips): Tom has 9 red chips and 3 blue chips, worth a total of 57 coins. $9R+3B=57$

Now we have a system of two linear equations. We want to find the value of B.

Step 1: Eliminate R To eliminate R, we can multiply the first equation by 9 and the second equation by 5 to make the R coefficients equal.

Multiply Equation 1 by 9: $9\times(5R+7B)=9\times69$ $45R+63B=621$ (Equation 3)

Multiply Equation 2 by 5: $5\times(9R+3B)=5\times57$ $45R+15B=285$ (Equation 4)

Step 2: Subtract Equation 4 from Equation 3 $(45R+63B)-(45R+15B)=621-285$

$45R-45R+63B-15B=336$ $48B=336$

Step 3: Solve for B B=48336 $B=7$

So, a single blue poker chip is worth 7 coins.

Optional Step 4: Find R (to verify) Substitute B=7 into either original equation. Let's use Equation 2: 9R+3B=57 9R+3(7)=57 9R+21=57 9R=57−21 9R=36 R=936 R=4

So, a single red poker chip is worth 4 coins.

Verification:

- Steve: 5(4)+7(7)=20+49=69 (Correct)
- Tom: 9(4)+3(7)=36+21=57 (Correct)

The final answer is 7.

ANS; 7

Problem 14:

Identify the base charge: The cameraman charged $76.35 for the first 100 photographs.

Calculate the amount charged for extra photographs: Total bill = $198.55 Amount for extra photos = Total bill - Base charge Amount for extra photos = 198.55−76.35=122.20

Calculate the number of extra photographs: Cost per extra photograph = $2.35 Number of extra photographs = Amount for extra photos / Cost per extra photograph Number of extra photographs = 122.20/2.35=52

Therefore, 52 extra photographs were taken.

The final answer is 52.

ANS: 52

Problem 15:

Initial Die State:

- Top: 3
- Front (forward along path): 5
- Back (opposite Front): 2

Rolling forward (4 rolls total to reach X): Each time the die rolls forward, the face that was on the "back" comes to the "top".

1. **Start:** Top is 3. Back is 2.

2. **Roll 1:** Old Back (2) becomes Top. Old Top (3) becomes Front. Old Front (5) becomes Bottom. New Back is 4 (opposite new Front 3). New Top: **2**
3. **Roll 2:** Old Back (4) becomes Top. Old Top (2) becomes Front. New Back is 5 (opposite new Front 2). New Top: **4**
4. **Roll 3:** Old Back (5) becomes Top. Old Top (4) becomes Front. New Back is 3 (opposite new Front 4). New Top: **5**
5. **Roll 4 (on X):** Old Back (3) becomes Top. Old Top (5) becomes Front. New Back is 2 (opposite new Front 5). New Top: **3**

The number on the top of the die when it rests on square X is 3.

The final answer is 3.

ANS: 3