# Solution:

## Problem 1:

The final number of silver tokens depends only on the number of white and black tokens left over at the end, not the order of the exchanges.

1. **Combined Trades:** A smart strategy is to combine the trades. For every 4 white and 3 black tokens Albert spends, he can efficiently gain 5 silver tokens. Starting with 75 of each color, he can do this 18 times. This gets him **90 silver tokens** and leaves him with **3 white** and **21 black** tokens.
2. **Final Exchanges:** With the remaining 3 white and 21 black tokens, he continues making any possible trades. After a series of exchanges at both booths, he is left with **1 white** and **2 black** tokens. These final trades give him another **13 silver tokens**.
3. **Total:** His total is 90+13=103 silver tokens. At this point, he can't make any more trades.

Correct answer: 103

## Problem 2:

**Understand the Setup** Let the three unknown prime numbers on the back of the cards be P1, P2, and P3. According to the problem, the sum of the numbers on each card is the same. Let's call this constant sum 'S'.

- Card 1: 32 + P1 = S
- Card 2: 30 + P2 = S
- Card 3: 59 + P3 = S

**Use Logic to Find the Sum (S)** Let's look at the numbers: 32 (even), 30 (even), and 59 (odd). The numbers on the back (P1, P2, P3) must all be prime.

The sum on the third card is 59 (odd) + P3 = S.

If P3 were any odd prime (like 3, 5, 7, etc.), the sum S would be odd + odd = even.

If S were even, then for the first card, 32 (even) + P1 = S (even), which means P1 must be an **even** prime. The only even prime is 2.

Similarly, for the second card, 30 (even) + P2 = S (even), which means P2 must also be an **even** prime, so P2 would also have to be 2.

This would make two of the numbers on the back identical (P1 = 2, P2 = 2), but the problem states all six numbers on the cards are **different**.

This means our initial assumption was wrong. The only way out is if P3 itself is the even prime, which is **2**.

- So, **P3 = 2**.

Now we can find the constant sum S using the third card: S = 59 + P3 = 59 + 2 = 61

2. **Find the Other Two Prime Numbers** Now that we know the sum for each card must be 61, we can find the other two prime numbers.
   - For the first card: 32 + P1 = 61 → P1 = 61 - 32 = 29
   - For the second card: 30 + P2 = 61 → P2 = 61 - 30 = 31

The numbers on the back of the cards are **29**, **31**, and **2**. These are all prime numbers, and all six numbers on the cards (32, 30, 59, 29, 31, 2) are different.

3. **Calculate the Final Sum** The question asks for the sum of the three numbers on the back of the cards. Sum = 29 + 31 + 2 = 62

The correct answer is **62**, **B**.

**Problem 3:**

**How to Solve It**

The key to this problem is to compare the average weight of the gold bars in each person's set.

1. **Summarize the Information**

   **Barbara:** Gets the 24 lightest bars, which are 45% of the total weight.

  **Monica:** Gets the 13 heaviest bars, which are 26% of the total weight.

  **Becky:** Gets the remaining bars in the middle. Let's say she gets x bars. The weight of her bars is 100% - 45% - 26% = 29% of the total.

2. **Calculate Average Weights** Let the total weight of all bars be W. We can find the average weight of the bars given to Barbara and Monica.

   **Barbara's average weight:** (45% of W) / 24 bars = 0.45W / 24 = 0.01875W

   **Monica's average weight:** (26% of W) / 13 bars = 0.26W / 13 = 0.02W

   **Becky's average weight:** (29% of W) / x bars = 0.29W / x

3. **Set Up an Inequality** Since Barbara has the lightest bars, Becky has the middle ones, and Monica has the heaviest, their average bar weights must be in that order: Average (Barbara) < Average (Becky) < Average (Monica)

Plugging in the values we calculated: $0.01875W < 0.29W / x < 0.02W$

We can cancel W from all parts of the inequality: $0.01875 < 0.29 / x < 0.02$

4. **Solve for x** We now have two separate inequalities to solve:

   $0.29 / x < 0.02 \rightarrow 0.29 < 0.02x \rightarrow 14.5 < x$

   $0.01875 < 0.29 / x \rightarrow 0.01875x < 0.29 \rightarrow x < 15.46...$

Combining these results, we get $14.5 < x < 15.46....$

Since the number of bars (x) must be a whole number, the only integer that fits this range is **15**.

The correct answer is **15**, **B**.

**Problem 4:**

**How to Solve It**

This problem can be solved by thinking about the sheep in terms of cost and profit. The 49 sheep that were left after the farmer recovered his initial cost represent the profit.

1. **Understand the Transaction**

The farmer's total cost is the price he paid for **749 sheep**.

He recovered this entire cost by selling just **700 sheep**.

This means his profit is the money he made from selling the remaining **49 sheep**.

2. **Calculate the Gain** The gain comes from the sale of the 49 extra sheep. The cost was covered by the first 700 sheep. So, the 49 sheep are the profit, measured against the cost base of the original 749 sheep.

We can express this as a simple fraction:

**Gain** = Value of 49 sheep

**Cost** = Value of 749 sheep (which is equal to the value of the 700 sheep he sold to break even)

3. **Find the Percentage** The percentage gain is the profit (the 49 sheep) divided by the original number of sheep that made up the cost (the 700 sheep that covered the cost), multiplied by 100.

Percent Gain = (Number of sheep representing profit / Number of sheep representing cost) * 100

Percent Gain = (49 / 700) * 100

Simplifying the fraction 49 / 700 gives 7 / 100.

Percent Gain = (7 / 100) * 100 = 7

The correct answer is **7**.

**Problem 5:**

Let x be the multiplier for the starting ratio (7:6:5) and y be the multiplier for the final ratio (6:5:4).

1. **Set Up Equations**

The total money at the start was 7x + 6x + 5x = 18x.

The total money at the end was 6y + 5y + 4y = 15y.

Since the total money is constant, 18x = 15y, which simplifies to 6x = 5y.

Arno's final money (6y) was $120 more than his starting money (7x), so 6y = 7x + 120.

2. **Solve for x** We can substitute 5y with 6x in the second equation. A quick way is to see that 6y = (6/5) * 5y.

6y = (6/5) * (6x) = (36/5)x

Now set the two expressions for 6y equal to each other: (36/5)x = 7x + 120

Multiply the entire equation by 5 to remove the fraction: 36x = 35x + 600

x = 600

3. **Calculate the Total** The total money at the start was 18x.

Total = 18 * 600 = 10,800

The correct answer is **10800**.

**Problem 6:**

It took at least **16 minutes** to fill the fish tank to 80% capacity.

### Volume Calculation

First, calculate the total volume of the fish tank and the amount of water needed.

- **Total Volume of the Tank:** The tank's volume is found by multiplying its dimensions: 60 cm × 30 cm × 40 cm = 72,000 $cm^3$
- **Volume of Water:** Pete filled the tank to 80% of its capacity. Since 1 $cm^3$ is equal to 1 ml, the volume of water is: 0.80 × 72,000 ml = 57,600 ml

### Flow Rate and Time

Next, determine the flow rate of the water and calculate the total time.

- **Flow Rate:** A 300 ml cup took *at least* 5 seconds to fill. To find the minimum time to fill the tank, we must use the fastest possible flow rate. The fastest rate occurs with the shortest time, which is exactly 5 seconds. Flow Rate = 300 ml / 5 seconds = 60 ml/second
- **Time to Fill:** Divide the required water volume by the flow rate to find the total time in seconds. Time = 57,600 ml / 60 ml/s = 960 seconds

- **Convert to Minutes:** Finally, convert the time from seconds to minutes. 960 seconds / 60 seconds/minute = 16 minutes

Correct answer: 16 minutes

**Problem 7:**

**Area Calculation** 📐

The problem states that the "area inside the circle but outside this triangle" equals the "area inside the triangle but outside the circle." This is a way of saying that the area of the circle is equal to the area of the triangle.

1. **Area of the Triangle:** The triangle is a right-angled triangle with sides 6, 8, and 10. The two shorter sides (6 and 8) serve as the base and height.
    - Area = (1/2) * base * height
    - Area = (1/2) * 6 * 8 = 24 square units.
2. **Area of the Circle:** Since the areas are equal, the area of the circle must also be **24** square units.

**Radius Calculation** ⭕

Now, we can use the formula for the area of a circle to find its radius.

- Area = $\pi r^2$
- 24 = 3.14 * $r^2$

To solve for the radius (r):

- $r^2$ = 24 / 3.14
- $r^2 \approx 7.6433$
- $r = \sqrt{7.6433}$
- $r \approx 2.76465$

Rounding to two decimal places, the radius is **2.76 units**.

**Problem 8:**

A quick way to find the area of a parallelogram when one vertex is at the origin (0,0) is to use the coordinates of the other two given vertices. This method, sometimes called the vector cross product or determinant method, works without needing to find the fourth vertex.

1. **Identify the Vectors** Let the vertices be O(0,0), A(1,4), and B(4,1). We can think of the two sides of the parallelogram connected to the origin as vectors.
    - Vector **OA** goes from (0,0) to (1,4), so its components are <1, 4>.
    - Vector **OB** goes from (0,0) to (4,1), so its components are <4, 1>.
2. **Calculate the Area** The area of the parallelogram formed by two vectors <a, b> and <c, d> is given by the absolute value of the expression ad - bc.
    - a = 1, b = 4
    - c = 4, d = 1

Area = | (1 * 1) - (4 * 4) | Area = | 1 - 16 | Area = | -15 | Area = **15**
The area of the parallelogram is **15** square units.

**Problem 9:**
To find the shaded area, you subtract the area of the three circles from the area of the large equilateral triangle.
**Area of the Three Circles** The area of one circle with a radius of 2 is $\pi r^2 = 3.14 * (2)^2 = 12.56$. The total area for all three is 3 * 12.56 = **37.68** square units.
**Area of the Triangle** For this specific arrangement, the area of the surrounding equilateral triangle is $24 + 16\sqrt{3}$. Using $\sqrt{3} \approx 1.732$, the area is $24 + 16(1.732) \approx$ **51.71** square units.
**Shaded Area** Subtract the circles' area from the triangle's area: 51.71 - 37.68 = **14.03** square units.
The correct answer is **14.03**.

**Problem 10:**
**Algebraic Solution**
Let the starting amounts for Adam, Brad, and Craig be A, B, and C. The total money is always $16 \times 3 = 48$ cents.

**After Adam's Turn:** Adam gives B to Brad and C to Craig. Brad's amount becomes 2B and Craig's becomes 2C.

**After Brad's Turn:** Brad doubles Adam's and Craig's current amounts. Craig's amount after Adam's turn (2C) is doubled again.

Final amount for Craig: $2 \times (2C) = 4C$.

We know this is 16, so $4C = 16$, which means **C = 4**.

**Find B and A:** Since Craig started with 4 cents and the total is 48, we know $A + B + 4 = 48$, which means $A + B = 44$.

After Adam's turn, his amount was $A - B - C$. This amount was then doubled by Brad to reach 16.

$2 \times (A - B - C) = 16$

$A - B - C = 8$

Since $C = 4$, we have $A - B - 4 = 8$, which means $A - B = 12$.

**Solve the System:** Now solve the two simple equations:

$A + B = 44$

$A - B = 12$ Adding them together gives $2A = 56$, so **A = 28**.

The correct answer is **28**.

**Problem 11:**

**Find the Total Number of Balls in Each Box**

First, we need to find the total number of balls (N) in each container. We know the total number of blue balls is 95.

- In **Box 1**, the ratio of red to blue is 9:1. This means 1/10 of the balls are blue.
  - Number of blue balls = (1/10) * N
- In **Box 2**, the ratio is 8:1. This means 1/9 of the balls are blue.
  - Number of blue balls = (1/9) * N

Now, set up an equation for the total number of blue balls: $(1/10)N + (1/9)N = 95$

To solve for N, find a common denominator (90): $(9/90)N + (10/90)N = 95$ $(19/90)N = 95$ $N = 95 * (90 / 19)$ $N = 5 * 90$ $N = 450$

So, there are **450 balls in each box**.

**Calculate the Number of Red Balls and Find the Difference**

Now we can find the number of red balls in each box and see the difference.

- **Red Balls in Box 1:** (9/10) * 450 = 9 * 45 = **405**
- **Red Balls in Box 2:** (8/9) * 450 = 8 * 50 = **400**

The difference is 405 - 400 = 5.

The correct answer is **5**.

**Problem 12:**

The sum of all possible four-digit numbers is **66,660**.

Each of the four digits (1, 2, 3, 4) will appear in each place value (ones, tens, hundreds, thousands) an equal number of times. With one digit fixed, the other three can be arranged in 3! = 6 ways, so each digit appears in each column 6 times.

The sum of the digits is 1 + 2 + 3 + 4 = 10.

Therefore, the sum of the values for each place value is:

- **Ones:** $6 \times 10 = 60$
- **Tens:** $6 \times 10 \times 10 = 600$
- **Hundreds:** $6 \times 10 \times 100 = 6{,}000$
- **Thousands:** $6 \times 10 \times 1{,}000 = 60{,}000$

Adding these together gives the total sum: 60,000 + 6,000 + 600 + 60 = 66,660.

A quicker calculation is $60 \times (1 + 10 + 100 + 1000) = 60 \times 1111 = 66{,}660$.

The correct answer is **66660**.

**Problem 13:**

Let the two positive integers be a and b. We can use the properties of the greatest common factor (GCD) to solve this problem.

The two numbers can be expressed as multiples of their GCD:

- a = 25x
- b = 25y Here, x and y are coprime integers (their only common factor is 1).

We know their sum is 350: 25x + 25y = 350 25(x + y) = 350 x + y = 350 / 25 = 14

We also know a key relationship: LCM = GCD × x × y. 825 = 25 × x × y x × y = 825 / 25 = 33

Now we just need to find two coprime numbers (x and y) that add up to 14 and multiply to 33. The two numbers are **3** and **11**.

Finally, we can find the original integers a and b:

- a = 25 × 3 = 75
- b = 25 × 11 = 275

The two numbers are 75 and 275. The larger of these is 275.

The correct answer is **275**.

**Problem 14:**

Let L_s be the initial length of the shorter candle and L_l be the initial length of the longer candle.

**Determine the Burn Rates**

First, we need to find the rate at which each candle burns.

- The **shorter candle** burns completely in 11 hours, so its burn rate is L_s / 11 units of length per hour.
- The **longer candle** burns completely in 7 hours, so its burn rate is L_l / 7 units of length per hour.

**Calculate Remaining Lengths**

Next, calculate the length of each candle remaining after burning for **3 hours**. The remaining length is the initial length minus the amount burned.

- **Shorter candle's remaining length:** L_s - 3 * (L_s / 11) = L_s * (1 - 3/11) = L_s * (8/11)
- **Longer candle's remaining length:** L_l - 3 * (L_l / 7) = L_l * (1 - 3/7) = L_l * (4/7)

**Find the Ratio**

The problem states that after three hours, both candles have the same length remaining. We can set their remaining lengths equal to each other to find the ratio.

$L\_s * (8/11) = L\_l * (4/7)$

To find the ratio of the shorter candle to the longer candle ($L\_s / L\_l$), we rearrange the equation:

$L\_s / L\_l = (4/7) / (8/11)$ $L\_s / L\_l = (4/7) * (11/8)$ $L\_s / L\_l = 44 / 56$

Simplifying the fraction by dividing the numerator and denominator by 4 gives:

$L\_s / L\_l = 11 / 14$

The correct answer is **11:14**.

**Problem 15:**

**Finding the Shortest Distance**

Let the dimensions of the box be length (L=4m), width (W=2m), and height (H=3m). The straight-line distance across the unfolded surfaces can be found using the Pythagorean theorem. We must calculate the three possible paths to find the shortest one.

- **Path 1:** Unfolding so the height and width are combined. Distance = $\sqrt{(L^2 + (H + W)^2)} = \sqrt{(4^2 + (3 + 2)^2)} = \sqrt{(16 + 25)} = \sqrt{41}$ meters
- **Path 2:** Unfolding so the length and width are combined. Distance = $\sqrt{(H^2 + (L + W)^2)} = \sqrt{(3^2 + (4 + 2)^2)} = \sqrt{(9 + 36)} = \sqrt{45}$ meters
- **Path 3:** Unfolding so the length and height are combined. Distance = $\sqrt{(W^2 + (L + H)^2)} = \sqrt{(2^2 + (4 + 3)^2)} = \sqrt{(4 + 49)} = \sqrt{53}$ meters

Comparing the three, the shortest path is **$\sqrt{41}$ meters**. $\sqrt{41} \approx 6.403$ meters.

**Calculating the Travel Time**

Now, we calculate the time it takes for the spider to travel this distance at its given speed. First, we must ensure our units are consistent.

- **Distance in centimeters:** 6.403 meters × 100 cm/m = 640.3 cm
- **Spider's speed:** 3 cm per second
- **Time:** Time = Distance / Speed Time = 640.3 cm / 3 cm/s ≈ 213.437 seconds

Rounding to the nearest hundredth, the time is **213.44 seconds**.