Solution:

Problem 1:

The Logic

This is a classic logic puzzle. We can solve it by starting with the one concrete fact we have and using the rules to eliminate possibilities.

Fact: Holmes wore a bow tie, which means he **wore a tie**.

Rule 2 states: Whenever he wears his tweed suit AND a purple shirt, he does NOT wear a tie.

Since he *did* wear a tie, we know he **could not** have been wearing the combination of a tweed suit and a purple shirt.

Rule 1 states: He always wears either a tweed suit OR sandals. Let's test the "tweed suit" possibility.

What if he wore the tweed suit?

From our first deduction, we know he can't be wearing a purple shirt with it. So, if he wore the tweed suit, he did **not** wear a purple shirt.

Now look at **Rule 3**: He never wears the tweed suit *unless* he is also wearing either a purple shirt OR sandals. Since he's not wearing a purple shirt, he **must** be wearing sandals.

But **Rule 4** says: Whenever he wears sandals, he also wears a purple shirt.

This creates a **contradiction**: He must wear a purple shirt (because of the sandals) and he must not wear a purple shirt (because of the tweed suit and tie).

Therefore, the initial assumption is wrong. **Holmes did not wear the tweed suit.**

Putting it all together:

Since he did not wear the tweed suit, he must have worn his other suit: the **black suit**.

According to Rule 1, since he didn't wear the tweed suit, he **must have worn sandals**.

According to Rule 4, since he wore sandals, he **must have worn a purple shirt**.

Conclusion

Yesterday, Holmes wore his **black suit**, a **purple shirt**, **sandals**, and a **bow tie**. Looking at the options, the pair that correctly describes what he wore is the black suit and purple shirt.

Correct answer: **D)**

Problem 2:

Let P be the amount Pete paid, J be Jason's ($100), and D be Dad's. Let T be the total cost.

From the statement "Pete paid one-third of what Jason and their dad paid together," we get:

$$P = \frac{1}{3}(J + D) => 3P = J + D$$

Since the total cost is T=P+J+D, we can substitute (J+D) with 3P:

$$T = P + 3P = 4P$$

This tells us Pete paid for exactly 1/4 of the book.

Now we use the other statement, "Pete and Jason contributed together was one-third of the final price":

$$P + J = \frac{1}{3}T$$

Substitute $P = \frac{1}{4}T$ and $J = 100$:

$$\frac{1}{4}T + 100 = \frac{1}{3}T$$

Solving for T:

$$100 = \frac{1}{12}T \implies T = 1200$$

Correct answer: **C)**

Problem 3:

Let N, L, and S be the number of normal, lame, and sitting ducks. The problem gives us three key pieces of information:

1. **Total Ducks:** N+L+S=99
2. **Total Legs:** 2N+L=100
3. **Duck Ratio:** S=1/2*(N+L)⟹2S=N+L

First, substitute the third relationship into the first equation:

(N+L)+S=99⟹(2S)+S=99

3S=99⟹S=33

There are 33 sitting ducks. This means the number of normal and lame ducks combined is N+L=99−33=66.

Now we have a simple system of two equations:

- 2N+L=100
- N+L=66

Subtracting the second equation from the first gives us **N = 34** (normal ducks). Finally, substituting N back into N+L=66 gives 34+L=66, which means **L = 32**.

Correct answer: **D)**

Problem 4:

Analyzing the Criteria

A three-digit number that contains at least one 0 and has at least one repeated digit must fit one of the following patterns (where *d* is a non-zero digit from 1 to 9):

1. **Two zeros:** The number has the form **d00**. (e.g., 100, 200)
2. **One zero and two other repeated digits:** The number has the form **dd0** or **d0d** (e.g., 110, 101)

Counting the Numbers

Now we check which of these numbers are divisible by 4. A number is divisible by 4 if the number formed by its last two digits is divisible by 4.

Case 1: Numbers of the form d00

The last two digits are "00". Since 0 is divisible by 4, all numbers of this form are divisible by 4. The possibilities for *d* are {1, 2, 3, 4, 5, 6, 7, 8, 9}. This gives us **9 numbers**: 100, 200, 300, 400, 500, 600, 700, 800, 900.

Case 2: Numbers of the form dd0

The last two digits form the number "d0". For "d0" to be divisible by 4, *d* must be an even digit. The possibilities for *d* are {2, 4, 6, 8}. This gives us **4 numbers**: 220, 440, 660, 880.

Case 3: Numbers of the form d0d

The last two digits form the number "0d". For "0d" to be divisible by 4, *d* must be divisible by 4. The possibilities for *d* are {4, 8}. This gives us **2 numbers**: 404, 808.

Total Count

Adding the counts from all three cases gives the total number of qualifying numbers:

Total=9+4+2=15

Correct answer: **A)**

Problem 5:

Vote Percentage Difference

Harold received **60%** of the votes, so Jacquie received the remaining **40%**. The difference in their vote shares is:

60%−40%=20%

Finding the Total

This **20%** difference in the vote share is equal to Harold's **24-vote** margin of victory. Let T be the total number of voters.

20% of T=24

0.20×T=24

To find the total number of voters, we can divide 24 by 0.20:

$$T = \frac{24}{0.2} = 120$$

A total of **120** people voted.

Correct answer: **E)**

Problem 6:

Setting up the Equations

Let n, d, and q be the number of 5-cent, 10-cent, and 25-cent coins, respectively. The problem provides three key pieces of information:

1. **Total Coins:** The sum of the coins is 32. $n+d+q=32$
2. **Total Value:** The total value of the coins is 390 cents. $5n+10d+25q=390$
3. **Coin Relationship:** There are 4 more 10-cent coins than 5-cent coins. $d=n+4$

Solving for the Number of Coins

We can solve this system of equations. First, substitute the third equation ($d=n+4$) into the first two to eliminate the variable d:

- Substituting into the **total coins** equation: $n+(n+4)+q=32 \Longrightarrow 2n + q = 28$
- Substituting into the **total value** equation (and dividing by 5 for simplicity): $n+2(n+4)+5q=78 \Longrightarrow 3n+8+5q=78 \Longrightarrow 3n + 5q = 70$

Now we have a simpler system with two variables. From the first simplified equation, we can express q as $q=28-2n$. We can substitute this into the second simplified equation:

There are **10** 5-cent coins.

The question asks for the number of 25-cent coins (q). We can now find it using the relationship we found earlier:

$q=28-2n$

$q=28-2(10)=8$

Marina has **8** 25-cent coins.

Correct answer: **B)**

Problem 7:

Solving the New Scenario

Now, we need to find the remainder when **3N** candies are divided by 7. We can find an expression for 3N by multiplying our first equation by 3:

$3N=3\times((7\times k)+3)$

$3N=(21\times k)+9$

When this new number of candies is divided by 7, the first part of the expression, **$21\times k$**, is perfectly divisible by 7 (since 21 is a multiple of 7). Therefore, the new remainder will only come from the second part, which is **9**.

To find the final remainder, we divide 9 by 7:

$9\div 7=1$ with a remainder of 2

So, 2 candies would have been left over.

Correct answer: **C)**

Problem 8:

Finding the Ending Odometer Reading

To find the greatest possible average speed, we must first find the greatest possible distance Karl could have traveled.

1. **Maximum Possible Distance:** Karl's speed never exceeded 80 km/h. In a 5-hour trip, the maximum distance he could have traveled is: $80\text{ km/h}\times 5\text{ h}=400\text{ km}$
2. **Maximum Odometer Reading:** The ending odometer reading must be a palindrome and cannot be more than 400 km past the starting point.

Starting reading: **13831**

Maximum possible reading: $13831+400=14231$

3. **Largest Palindrome:** We need to find the largest palindrome between 13831 and 14231.

The next palindrome after 13831 is 13931.

The next palindrome after that starts with "14...". A 5-digit palindrome starting with 14 must be of the form 14c41.

- If c=0, we get 14041. (This is valid as it's < 14231)
- If c=1, we get **14141**. (This is also valid as it's < 14231)
- If c=2, we get 14241. (This is too high)

The largest possible palindrome for the ending reading is **14141**.

Calculating the Average Speed

Now we can calculate the distance traveled and the average speed.

- **Distance Traveled:** 14141 km−13831 km=310 km
- **Average Speed:** $\frac{Distance}{Time} = \frac{310\ km}{5\ hours} = 62\ km/h$

Correct answer: **A)**

Problem 9:

Finding the Distance to Home

First, let's determine the distance from school to Karl's home.

The difference in arrival times between the two scenarios is from 4:30 PM to 5:15 PM, which is **45 minutes**, or **0.75 hours**. This time difference is caused solely by the difference in his cycling speed.

Let **D** be the distance to home. The travel time is Distance / Speed. We can set up an equation based on the difference in travel times:

Equation Setup

(Time at 10 km/h)−(Time at 20 km/h)=0.75 hours

This first line states that the difference in travel time between the two scenarios is 45 minutes (or 0.75 hours).

Solving for Distance (D)

$$\frac{D}{10} - \frac{D}{20} = 0.75$$

This step replaces the travel times with the expression Distance / Speed.

$$\frac{2D - D}{20} = 0.75$$

Finally, the equation is solved for D by multiplying both sides by 20, showing that the distance is **15 km**.

Calculating the Required Speed

Now we can find the speed needed to arrive at 5:00 PM.

1. **Find the departure time:** In the first scenario, Karl travels 15 km at 20 km/h

- Travel time=$\frac{15\ km}{20\ km/h} = 0.75\ hours = 45\ minutes.$
- If he arrives at 4:30 PM after a 45-minute trip, he must leave school at **3:45 PM.**

2. **Find the required speed:** To arrive at 5:00 PM, his travel time must be:

- 5:00 PM - 3:45 PM = 1 hour and 15 minutes = **1.25 hours**.
- The required speed is Distance / Time.
- Speed=$\frac{15\ km}{1.25\ hours} = 12\ km/h$

Correct answer: **C)**

Problem 10:

The volume of the original 3x3x3 cube is **27 cm³**. Since the smaller cubes are not all identical and have integer edges, they must be a mix of 1x1x1 cubes (volume 1) and 2x2x2 cubes (volume 8).

A 3x3x3 cube can physically fit only **one** 2x2x2 cube inside it.

- **One 2x2x2 cube** accounts for **8 cm³** of the volume.
- The **remaining volume** is 27−8=19 cm³.
- This remaining volume must be filled by **19** of the 1x1x1 cubes.

The total number of smaller cubes is 1+19=20.

Correct answer: **E)**

Problem 11:

The sum of the four numbers along a side is **21**.

The Key Insight

To solve this "magic triangle" puzzle, we can find a relationship between the sum of all the numbers and the sum along each side.

Let **S** be the sum of the numbers along one side. If we add the sums of all three sides together (3S), we are counting every number in the triangle. The numbers in the corners are counted twice (since they are part of two sides), and the numbers in the middle are counted once.

Therefore, the sum of the three sides is equal to the sum of all nine numbers plus the sum of the three corner numbers.

3S=(Sum of all numbers)+(Sum of corner numbers)

Finding the Sum of the Corners

1. **Sum of all numbers:** The sum of the integers from 1 to 9 is: $1+2+3+4+5+6+7+8+9=45$.
2. **Sum of corner numbers:** We know two corners are **6** and **9**. Let the third corner be **C**. The sum of the corners is $6+9+C=15+C$.

Plugging these into our main equation:

$3S=45+(15+C) \Longrightarrow 3S=60+C$

This means that **60 + C** must be a number divisible by 3. Since 60 is already divisible by 3, the value of the third corner, **C**, must also be divisible by 3. The numbers 6 and 9 are already used. The remaining numbers available for the third corner are {1, 2, 3, 4, 5, 7, 8}. The only one of these that is divisible by 3 is **3**. So, the third corner must be **3**.

Calculating the Final Sum

Now we know the sum of the three corners is $6+9+3=18$. We can solve for S:

Correct answer: **C)**

Problem 12:

The question asks for the position of the number 53 when counting backwards from 201 to 3. This is simply the count of how many numbers there are in the list from 201 down to 53, inclusive.

To find the number of integers in an inclusive range, you can use the formula:

$\text{Number of items}=\text{Last}-\text{First}+1$

Applying this to our backward count:

$n=201-53+1$

$n=148+1=149$

Therefore, 53 is the **149th** number counted when counting backwards from 201. The information about the forward count is not needed for this direct calculation.

Correct answer: **A)**

Problem 13:

Finding the Dimensions

The length, width, and height of the prism form an arithmetic sequence with a common difference of 3. We can represent these dimensions algebraically as:

- $x-3$
- x
- $x+3$

The volume of a rectangular prism is found by multiplying its three dimensions. We're given that the volume is 440 cm^3.

$(x-3)(x)(x+3)=440$

$x(x2-9)=440$

Instead of solving this cubic equation algebraically, we can test integer values for x. Since x3 will be slightly larger than 440, we can estimate x to be close to 3440 (which is between 7 and 8).

Let's test **x = 8**:

$(8-3)(8)(8+3)=5\times8\times11=40\times11=440$

This is a perfect match. So, the dimensions of the prism are **5 cm**, **8 cm**, and **11 cm**.

Calculating the Surface Area

The formula for the surface area of a rectangular prism is 2(LW+LH+WH).

Correct answer: **D)**

Problem 14:

Finding the Center of the Circle

The problem states that the circle is tangent to the y-axis at the point **(0, 2)**. A radius to the point of tangency is always perpendicular to the tangent line. Since the y-axis is a vertical line, the radius to the point (0, 2) must be a horizontal line.

This tells us two things about the circle's center, (h, k):

1. The y-coordinate of the center must be **k = 2**.

2. The distance from the center to the y-axis is the radius, **r**. This distance is equal to the x-coordinate of the center, so **h = r**.

Therefore, the center of the circle is at **(r, 2)**.

Using the Circle's Equation

The standard equation for a circle is $(x-h)^2+(y-k)^2=r^2$. Substituting the center we found, (r, 2), the equation becomes:

$$(x-r)^2+(y-2)^2=r^2$$

We know the circle passes through the x-intercept **(8, 0)**. We can plug these x and y values into the equation to solve for r:

$$(8-r)^2+(0-2)^2=r^2$$

Now, we expand and solve the equation:

Correct answer: **B)**

Problem 15:

Trapezoid Area Method

Here is an alternative method. The quadrilateral **MARE** is a **trapezoid**, because sides MA and RE are parallel (since they are parts of opposite sides of the square).

The formula for the area of a trapezoid is:

$$Area=\frac{1}{2}\times(\text{base}_1+\text{base}_2)\times\text{height}$$

Finding the Dimensions

First, we need to find the dimensions of the trapezoid. Let the side length of the square be **s**.

- **Height:** The height of the trapezoid is the side of the square, **RA = s**.
- **Base 1:** The bottom base is side **MA = s**.
- **Base 2:** The top base is **RE**. We can find its length by subtracting EY from the full side RY: **RE = s - 12**.

To find the value of **s**, we use trigonometry in the right-angled triangle MEY:

Calculating the Area

Now we have the values for the height and both bases:

Height ≈14.301

Base 1 ≈14.301

Base 2 ≈14.301−12=2.301

Plugging these into the trapezoid area formula:

This result rounds to **119**.

Correct answer: **A)**