ARCHIMEDES MATH COMPETITION

Second Round 2021-2022

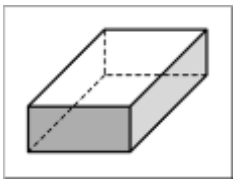
Grade 9

Problem №1.

If the radius of a circle is increased by 1 unit, what is the ratio of the new circumference to the new diameter?
(Write your answer in terms of π)

Problem №2.

Yuki draws the 3D image of a rectangular prism, similar to the one shown.



She then labels the vertices from A to H and starts listing pairs of parallel edges.

How many such pairs can she list?

Problem №3.

Find the number of ordered pairs of integers (a, b) such that the sequence $3, 4, 5, a, b, 30, 40, 50$ is strictly increasing and no set of four (not necessarily consecutive) terms forms an arithmetic progression.

Problem №4.

Let $a, b, c, d, e, f, g, h, i$ be distinct integers from 1 to 9. The minimum possible positive value of $\frac{a \cdot b \cdot c - d \cdot e \cdot f}{g \cdot h \cdot i}$ can be written as $\frac{m}{n}$ where m and n are relatively prime positive integers. Find $m + n$.

Problem №5.

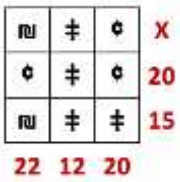
Adults made up $\frac{5}{12}$ of the crowd of people at a concert. After a bus carrying 50 more people arrived, adults made up $\frac{11}{25}$ of the people at the concert. Find the minimum number of adults who could have been at the concert after the bus arrived.

Problem №6.

Twenty distinct points are marked on a circle and labeled 1 through 20 in clockwise order. A line segment is drawn between every pair of points whose labels differ by a prime number. Find the number of triangles formed whose vertices are among the original 20 points.

Problem №7.

In the 3 × 3 grid shown, each of the three symbols has a different value. The sum of the values of the symbols in each row is given to the right of that row, and the sum of the values of the symbols in each column is given below that column.



What is the value of **X**?

Problem №8.

Adam, Josh, Sam, Mike, and Trevor are watching a movie in the cinema from a special VIP booth, that has five seats in a row. Adam is not seated next to Josh. Sam is seated next to Mike.

Which one of the five friends ***cannot*** be seated in the **middle seat**?