Second Round 2021-2022

Grade 11

Problem №1.

Andrew's lawn is twice as big as Bert's and three times as big as Curt's lawn. Curt's lawn mower cuts half as fast as Bert's and one third as fast as Andrew's lawn mower. If they all start mowing their own lawns at the same time, who will finish first, Andrew, Bert, or Curt?

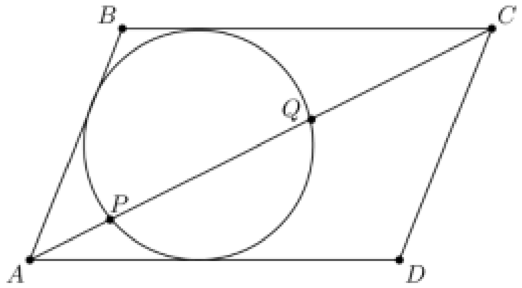
Problem №2.

Three spheres with radii 11, 13, and 19 are mutually externally tangent. A plane intersects the spheres in three congruent circles centered at A, B, and C, respectively, and the centers of the spheres all lie on the same side of this plane. Suppose that $AB^2 = 560$. Find AC^2.

Problem №3.

Let $ABCD$ be a parallelogram with $\angle BAD < 90°$. A circle tangent to sides $\overline{DA}$, $\overline{AB}$, and $\overline{BC}$ intersects diagonal $\overline{AC}$ at points P and Q with $AP < AQ$, as shown.

Suppose that $AP = 3$, $PQ = 9$, and $QC = 16$. Then the area of $ABCD$ can be expressed in the form $m\sqrt{n}$, where m and n are positive integers, and n is not divisible by the square of any prime. Find $m + n$.



Problem №4.

Azar, Carl, Jon, and Sergey are the four players left in a singles tennis tournament. They are randomly assigned opponents in the semifinal matches, and the winners of those matches play each other in the final match to determine the winner of the tournament. When Azar plays Carl, Azar will win the match with probability $\frac{2}{3}$. When either Azar or Carl plays either Jon or Sergey, Azar or Carl will win the match with probability $\frac{3}{4}$. Assume that outcomes of different matches are independent. The probability that Carl will win the tournament is $\frac{p}{q}$, where p and q are relatively prime positive integers. Find $p + q$.

Problem №5.

Find the remainder when $\binom{\binom{3}{2}}{2} + \binom{\binom{4}{2}}{2} + \cdots + \binom{\binom{40}{2}}{2}$ is divided by 1000.

Problem №6.

Let $x_1 \leq x_2 \leq \cdots \leq x_{100}$ be real numbers such that $|x_1| + |x_2| + \cdots + |x_{100}| = 1$ and $x_1 + x_2 + \cdots + x_{100} = 0$. Among all such 100-tuples of numbers, the greatest value that $x_{76} - x_{16}$ can achieve is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.