Grade 10

Problem №1.

Find the smallest number that when divided by 11 gives a remainder of 10, when divided by 10 gives a remainder of 9, when divided by 9, gives a remainder of 8, when divided by 8 gives a remainder of 7, and so forth, down to a remainder of 1 when the number is divided by 2.

Problem №2.

Adam, Josh, Sam, Mike, and Trevor are watching a movie in the cinema from a special VIP booth, that has five seats in a row. Adam is not seated next to Josh. Sam is seated next to Mike.

Which one of the five friends ***cannot*** be seated in the **middle seat**?

Problem №3.

Quadratic polynomials $P(x)$ and $Q(x)$ have leading coefficients of 2 and -2, respectively. The graphs of both polynomials pass through the two points $(16, 54)$ and $(20, 53)$. Find $P(0) + Q(0)$.

Problem №4.

In isosceles trapezoid $ABCD$, parallel bases $\overline{AB}$ and $\overline{CD}$ have lengths 500 and 650, respectively, and $AD = BC = 333$. The angle bisectors of $\angle A$ and $\angle D$ meet at P, and the angle bisectors of $\angle B$ and $\angle C$ meet at Q. Find PQ.

Problem №5.

Ellina has twelve blocks, two each of
red (**R**), blue (**B**), yellow (**Y**), green (**G**), orange (**O**) and purple (**P**). Call an arrangement of blocks even if there is an even number of blocks between each pair of blocks of the same color. For example, the arrangement:

R B B Y G G Y R O P P O

Ellina arranges her blocks in a row in random order. The probability that her arrangement is even is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

Problem №6.

Let $x_1 \leq x_2 \leq \cdots \leq x_{100}$ be real numbers such that

$|x_1| + |x_2| + \cdots + |x_{100}| = 1$ and $x_1 + x_2 + \cdots + x_{100} = 0$. Among all such 100-tuples of numbers, the greatest value that $x_{76} - x_{16}$ can achieve is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.